

PEARSON

Math Makes Sense

9

Preparation and Practice Book

Reproducible Version

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About *Math Makes Sense 9* Preparation and Practice Book

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Welcome to *Math Makes Sense 9*.

These pages describe how this *Preparation and Practice Book* can help to support your success this year.

For the main lessons of *Math Makes Sense 9*, this workbook gives you:

- ***Skill Builder pages***

- Each topic identifies a prerequisite skill or concept you will need in that lesson.
- Each topic starts with a quick review and a short example.
- General rules are highlighted to help you find them more easily.
- Try the **Check** questions to see whether you're ready to move on.




- ***Matching lessons for the ones in your Student Book***

- General rules are highlighted to help you find them more easily.
- **Examples** show you what you need to know and be able to do.
- For each Example, **Check** lets you try a question on your own before moving on.
- **Practice** questions help you put it all together during the lesson.

- ***Step-by-step support***

- Every question supports your work by breaking down the steps you need to follow to arrive at a correct solution.

For each unit of *Math Makes Sense 9*, you'll find:

- an opening page that tells you **What You'll Learn** and **Why It's Important**, as well as an expanded list of **Key Words**
 - **Checkpoint** pages with mid-unit review
 - a **Unit Puzzle** for fun and reinforcement
 - a **Unit Study Guide** to summarize key ideas
 - **Unit Review** pages to help you see what you remember
- 

Square Roots and Surface Area

What You'll Learn

- Find square roots of fractions and decimals that are perfect squares.
- Approximate the square roots of fractions and decimals that are not perfect squares.
- Find the surface areas of composite objects.

Why It's Important

Square roots are used by

- police officers, to estimate the speed of a vehicle when it crashed
- vets, to calculate drug dosages

Surface area is used by

- painters, to find the number of cans of paint needed to paint a room
- farmers, to find the amount of fertilizer needed for a field

Key Words

square
square root
perfect square
non-perfect square
terminating decimal

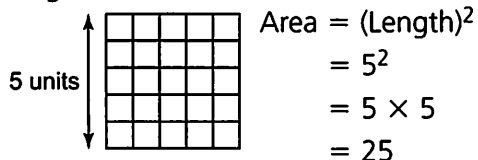
repeating decimal
non-terminating,
non-repeating decimal
surface area
composite object

1.1 Skill Builder

Side Lengths and Areas of Squares

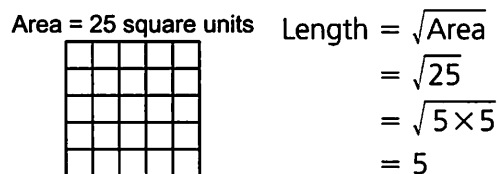
The side length and area of a square are related.

- The area is the **square** of the side length.



The area is 25 square units.

- The side length is the **square root** of the area.



The side length is 5 units.

Check

1. Which square and square root are modelled by each diagram?

Diagram	Square Modelled	Square Root Modelled
a)	$(\text{Length})^2 = \text{Area}$ $7^2 = \underline{\quad}$ The area is 49 square units.	$\sqrt{\text{Area}} = \text{Length}$ $\sqrt{49} = \underline{\quad}$ The side length is 7 units.
b)	$\underline{\quad} = \underline{\quad}$ The area is $\underline{\quad}$ square units.	$\sqrt{\underline{\quad}} = \underline{\quad}$ The side length is $\underline{\quad}$ units.
c)	$\underline{\quad} = \underline{\quad}$ The area is $\underline{\quad}$ square units.	$\sqrt{\underline{\quad}} = \underline{\quad}$ The side length is $\underline{\quad}$ units.
d)	$\underline{\quad} = \underline{\quad}$ The area is $\underline{\quad}$ square units.	$\sqrt{\underline{\quad}} = \underline{\quad}$ The side length is $\underline{\quad}$ units.

Whole Number Squares and Square Roots

- The square of a number is the number multiplied by itself. $5^2 = 5 \times 5 = 25$
- A square root of a number is one of 2 equal factors of the number. $\sqrt{25} = \sqrt{5 \times 5} = 5$
- Squaring and taking a square root are inverse operations. $5^2 = 25$ and $\sqrt{25} = 5$

Check

1. Complete each sentence.

- a) $4^2 = 16$, so $\sqrt{16} = \underline{\quad}$ b) $12^2 = \underline{\quad}$, so $\sqrt{\underline{\quad}} = \underline{\quad}$
 c) $\sqrt{25} = \underline{\quad}$, since $\underline{\quad} = 25$ d) $\sqrt{100} = \underline{\quad}$, since $\underline{\quad} = \underline{\quad}$

Perfect Squares

A number is a **perfect square** if it is the product of 2 equal factors.

25 is a perfect square because $25 = 5 \times 5$.

24 is a **non-perfect square**. It is not the product of 2 equal factors.

Check

1. Complete each sentence.

First 12 Whole-Number Perfect Squares			
Perfect Square	Square Root	Perfect Square	Square Root
$1^2 = 1 \times 1 = 1$	$\sqrt{1} = 1$	$7^2 = \underline{\quad} \times \underline{\quad} = \underline{\quad}$	$\sqrt{\underline{\quad}} = \underline{\quad}$
$2^2 = 2 \times 2 = 4$	$\sqrt{4} = 2$	$8^2 = \underline{\quad} \times \underline{\quad} = \underline{\quad}$	$\sqrt{\underline{\quad}} = \underline{\quad}$
$3^2 = \underline{\quad} \times \underline{\quad} = \underline{\quad}$	$\sqrt{\underline{\quad}} = \underline{\quad}$	$9^2 = \underline{\quad} \times \underline{\quad} = \underline{\quad}$	$\sqrt{\underline{\quad}} = \underline{\quad}$
$4^2 = \underline{\quad} \times \underline{\quad} = \underline{\quad}$	$\sqrt{\underline{\quad}} = \underline{\quad}$	$10^2 = \underline{\quad} \times \underline{\quad} = \underline{\quad}$	$\sqrt{\underline{\quad}} = \underline{\quad}$
$5^2 = \underline{\quad} \times \underline{\quad} = \underline{\quad}$	$\sqrt{\underline{\quad}} = \underline{\quad}$	$11^2 = \underline{\quad} \times \underline{\quad} = \underline{\quad}$	$\sqrt{\underline{\quad}} = \underline{\quad}$
$6^2 = \underline{\quad} \times \underline{\quad} = \underline{\quad}$	$\sqrt{\underline{\quad}} = \underline{\quad}$	$12^2 = \underline{\quad} \times \underline{\quad} = \underline{\quad}$	$\sqrt{\underline{\quad}} = \underline{\quad}$

1.1 Square Roots of Perfect Squares

FOCUS Find the square roots of decimals and fractions that are perfect squares.

The square of a fraction or decimal is the number multiplied by itself.

$$\begin{aligned}\left(\frac{2}{3}\right)^2 &= \frac{2}{3} \times \frac{2}{3} \\ &= \frac{2 \times 2}{3 \times 3} \\ &= \frac{4}{9}\end{aligned}$$

$$\begin{aligned}(1.5)^2 &= 1.5 \times 1.5 \\ &= 2.25\end{aligned}$$

$\frac{4}{9}$ and 2.25 are perfect squares because they are the product of 2 equal factors.

$$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}, \text{ so}$$

$\frac{2}{3}$ is a square root of $\frac{4}{9}$.

$$\text{We write: } \sqrt{\frac{4}{9}} = \frac{2}{3}$$

$$2.25 = 1.5 \times 1.5, \text{ so}$$

1.5 is a square root of 2.25.

$$\text{We write: } \sqrt{2.25} = 1.5$$

Each equal factor is a square root of the perfect square.

Example 1 Finding a Perfect Square Given Its Square Root

Calculate the number whose square root is:

a) $\frac{5}{8}$

b) 1.2

Solution

A square root of a number is one of two equal factors of the number.

a) $\frac{5}{8}$

$$\begin{aligned}\frac{5}{8} \times \frac{5}{8} &= \frac{5 \times 5}{8 \times 8} \\ &= \frac{25}{64}\end{aligned}$$

So, $\frac{5}{8}$ is a square root of $\frac{25}{64}$.

b) 1.2

Use a calculator.

$$1.2 \times 1.2 = 1.44$$

So, 1.2 is a square root of 1.44.

Check

1. Calculate the perfect square with the given square root.

a) $\frac{3}{8}$

$$\frac{3}{8} \times \frac{3}{8} = \frac{\quad \times}{\quad \times \quad}$$

$$= \underline{\hspace{2cm}}$$

$\frac{3}{8}$ is a square root of $\underline{\hspace{2cm}}$.

b) $\frac{3}{2}$

$$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

$\frac{3}{2}$ is a square root of $\underline{\hspace{2cm}}$.

c) 0.5

$$0.5 \times 0.5 = \underline{\hspace{2cm}}$$

0.5 is a square root of $\underline{\hspace{2cm}}$.

d) 2.5

$$2.5 \times 2.5 = \underline{\hspace{2cm}}$$

2.5 is a square root of $\underline{\hspace{2cm}}$.

Example 2 Identifying Fractions that Are Perfect Squares

Is each fraction a perfect square? If so, find its square root.

a) $\frac{16}{25}$

b) $\frac{9}{20}$

Solution

Check if the numerator and denominator are perfect squares.

a) $\frac{16}{25}$

16 = 4 × 4, so 16 is a perfect square.

25 = 5 × 5, so 25 is a perfect square.

So, $\frac{16}{25}$ is a perfect square.

b) $\frac{9}{20}$

9 = 3 × 3, so 9 is a perfect square.

20 is not a perfect square.

So, $\frac{9}{20}$ is not a perfect square.

Check

1. Determine whether the fraction is or is not a perfect square. How do you know?

a) $\frac{9}{49}$

9 _____ a perfect square because _____.

49 _____ a perfect square because _____.

So, $\frac{9}{49}$ _____ a perfect square.

b) $\frac{25}{13}$

25 _____ a perfect square because _____.

13 _____ a perfect square because _____.

So, $\frac{25}{13}$ _____ a perfect square.

- c) $\frac{64}{81}$ 64 _____ a perfect square because _____
 81 _____ a perfect square because _____
 So, $\frac{64}{81}$ _____ a perfect square.

2. Find the value of each square root.

a) $\sqrt{\frac{9}{4}} = \sqrt{\frac{\quad \times \quad}{\quad \times \quad}} = \frac{\quad}{\quad}$

b) $\sqrt{\frac{16}{81}} = \sqrt{\frac{\quad \times \quad}{\quad \times \quad}} = \frac{\quad}{\quad}$

A **terminating decimal** ends after a certain number of decimal places.

A **repeating decimal** has a repeating pattern of digits in the decimal expansion.

The bar shows the digits that repeat.

Terminating	Repeating	Non-terminating and non-repeating
0.5 0.28	0.333 333 ... = $0.\overline{3}$	1.414 213 56 ... 7.071 067 812 ...
	0.191 919 ... = $0.\overline{19}$	

You can use a calculator to find out if a decimal is a perfect square.

The square root of a perfect square decimal is either a terminating decimal or a repeating decimal.

Example 3 Identifying Decimals that Are Perfect Squares

Is each decimal a perfect square? How do you know?

a) 1.69

b) 3.5

Solution

Use a calculator to find the square root of each number.

a) $\sqrt{1.69} = 1.3$

The square root is the terminating decimal 1.3.

So, 1.69 is a perfect square.

b) $\sqrt{3.5} \doteq 1.870\ 828\ 693$

The square root appears to be a decimal that neither repeats nor terminates.

So, 3.5 is not a perfect square.

The symbol \doteq means "approximately equal to".

Check

1. Complete the table to find whether each decimal is a perfect square.
The first one is done for you.

	Decimal	Value of square root	Type of decimal	Is decimal a perfect square?
a)	70.5	8.396 427 811 ...	Non-repeating Non-terminating	No
b)	5.76	_____	_____	_____
c)	0.25	_____	_____	_____
d)	2.5	_____	_____	_____

Practice

1. Calculate the number whose square root is:

a) $\frac{1}{4}$

$$\frac{1}{4} \times \frac{1}{4} = \frac{\quad \times \quad}{\quad \times \quad}$$

$$= \frac{\quad}{\quad}$$

$\frac{1}{4}$ is a square root of _____.

b) $\frac{2}{7}$

$$\frac{2}{7} \times \frac{2}{7} = \frac{\quad \times \quad}{\quad \times \quad}$$

$$= \frac{\quad}{\quad}$$

$\frac{2}{7}$ is a square root of _____.

c) 0.6

$$\frac{\quad}{\quad} \times \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

0.6 is a square root of _____.

d) 1.1

$$\frac{\quad}{\quad} \times \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

1.1 is a square root of _____.

2. Identify the fractions that are perfect squares. The first one has been done for you.

	Fraction	Is numerator a perfect square?	Is denominator a perfect square?	Is fraction a perfect square?
a)	$\frac{81}{125}$	Yes; $9 \times 9 = 81$	No	No
b)	$\frac{25}{49}$	_____	_____	_____
c)	$\frac{36}{121}$	_____	_____	_____
d)	$\frac{17}{25}$	_____	_____	_____
e)	$\frac{9}{100}$	_____	_____	_____

3. Find each square root.

a) $\sqrt{\frac{49}{100}} = \sqrt{\frac{\quad \times \quad}{\quad \times \quad}}$
= _____

b) $\sqrt{\frac{25}{144}} = \sqrt{\frac{\quad \times \quad}{\quad \times \quad}}$
= _____

c) $\sqrt{\frac{1}{16}} = \sqrt{\frac{\quad \times \quad}{\quad \times \quad}}$
= _____

d) $\sqrt{\frac{9}{400}} = \sqrt{\frac{\quad \times \quad}{\quad \times \quad}}$
= _____

4. Use a calculator. Find each square root.

a) $\sqrt{8.41} = \underline{\hspace{2cm}}$ b) $\sqrt{0.0676} = \underline{\hspace{2cm}}$ c) $\sqrt{51.125} = \underline{\hspace{2cm}}$ d) $\sqrt{6.25} = \underline{\hspace{2cm}}$

5. Which decimals are perfect squares?

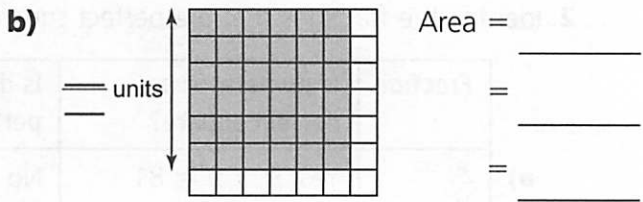
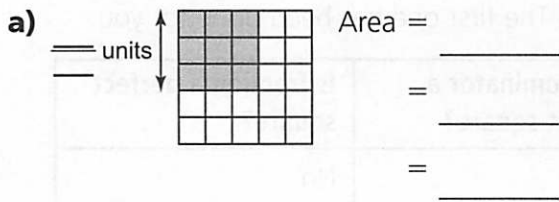
a) 1.44 $\sqrt{1.44} = \underline{\hspace{2cm}}$
The square root is a decimal that _____.
So, 1.44 _____ a perfect square.

b) 30.25 $\sqrt{30.25} = \underline{\hspace{2cm}}$
The square root is a decimal that _____.
So, 30.25 _____ a perfect square.

c) 8.5 $\sqrt{8.5} \doteq \underline{\hspace{2cm}}$
The square root is a decimal that _____.
So, 8.5 _____ a perfect square.

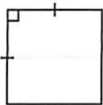
d) 0.0256 $\sqrt{0.0256} = \underline{\hspace{2cm}}$
The square root is a decimal that _____.
So, 0.0256 _____ a perfect square.


6. Find the area of each square.



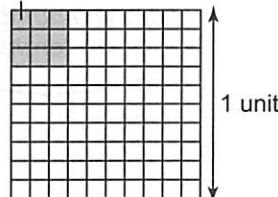
Area = (Length)²

The area is _____

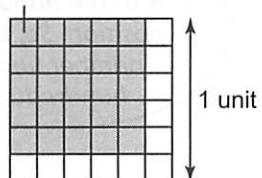
c)  Area = _____
 = _____ × _____
 = _____

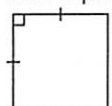
d)  Area = _____
 = _____ × _____
 = _____

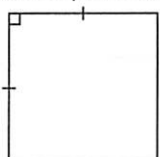
7. Find the side length of each square.

a) Area = $\frac{9}{100}$ square units Side Length = $\sqrt{\quad}$ Length = $\sqrt{\text{Area}}$

 = $\sqrt{\quad}$
 = _____

The side length is _____ units.

b) Area = $\frac{25}{36}$ square units Length = $\sqrt{\quad}$

 = $\sqrt{\quad}$
 = _____

c) Area = 0.01 square units Length = $\sqrt{\quad}$

 = _____

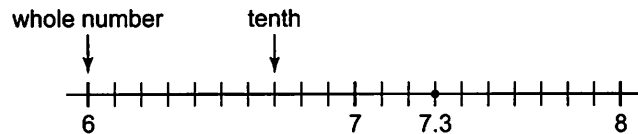
d) Area = 46.24 square units Length = $\sqrt{\quad}$

 = _____

1.2 Skill Builder

Degree of Accuracy

We are often asked to write an answer to a given decimal place.
To do this, we can use a number line.

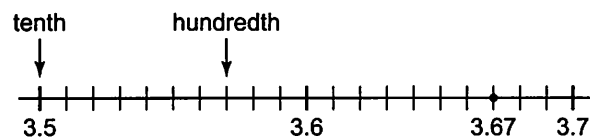
To write 7.3 to the nearest whole number:
Place 7.3 on a number line in tenths.



7.3 is closer to 7 than to 8.
So, 7.3 to the nearest whole number is: 7

3 is the last digit. It is in the tenths position. So, use a number line in tenths.

To write 3.67 to the nearest tenth:
Place 3.67 on a number line in hundredths.



3.67 is closer to 3.7 than to 3.6.
So, 3.67 to the nearest tenth is: 3.7

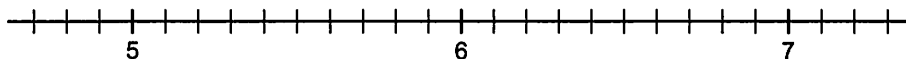
7 is the last digit. It is in the hundredths position. So, use a number line in hundredths.

Check

1. Write each number to the nearest whole number.

Mark it on the number line.

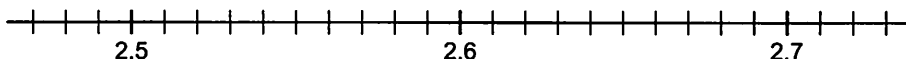
- a) 5.3 b) 6.8 c) 7.1 d) 6.4



2. Write each number to the nearest tenth.

Mark it on the number line.

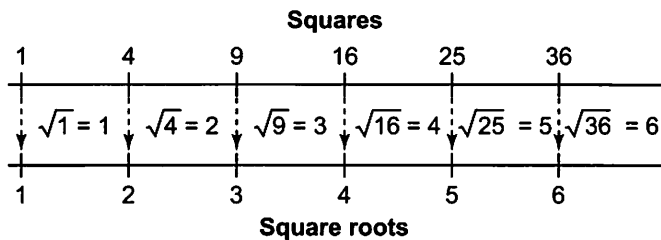
- a) 2.53 b) 2.64 c) 2.58 d) 2.66



Squares and Square Roots on Number Lines

Most numbers are not perfect squares.

You can use number lines to estimate the square roots of these numbers.



10 is between the perfect squares 9 and 16.

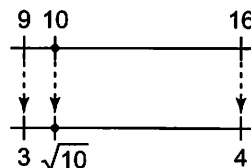
So, $\sqrt{10}$ is between $\sqrt{9}$ and $\sqrt{16}$.

$\sqrt{9} = 3$ and $\sqrt{16} = 4$

So, $\sqrt{10}$ is between 3 and 4.

Check with a calculator.

$\sqrt{10} \approx 3.2$, which is between 3 and 4.



10 is closer to 9 than 16, so $\sqrt{10}$ is closer to 3 than 4.

Check

1. Between which 2 consecutive whole numbers is each square root?

Explain.

a) $\sqrt{22}$

22 is between the perfect squares 16 and 25.

So, $\sqrt{22}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$.

$\sqrt{\quad} = \quad$ and $\sqrt{\quad} = \quad$

So, $\sqrt{22}$ is between \quad and \quad .

b) $\sqrt{6}$

6 is between the perfect squares \quad and \quad .

So, $\sqrt{6}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$.

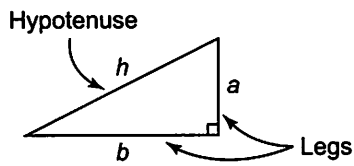
$\sqrt{\quad} = \quad$ and $\sqrt{\quad} = \quad$

So, $\sqrt{6}$ is between \quad and \quad .

Refer to the squares and square roots number lines.

The Pythagorean Theorem

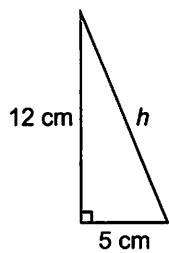
You can use the Pythagorean Theorem to find unknown lengths in right triangles.



Pythagorean Theorem

$$h^2 = a^2 + b^2$$

To find the length of the hypotenuse, h , in this triangle:



$$h^2 = 5^2 + 12^2$$

$$h^2 = 25 + 144$$

$$h^2 = 169$$

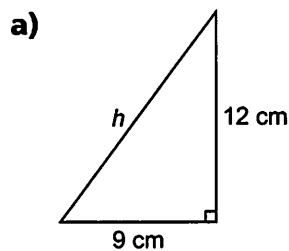
$$h = \sqrt{169}$$

$$h = 13$$

The length of the hypotenuse is 13 cm.

Check

1. Use the Pythagorean Theorem to find the length of each hypotenuse, h .



$$h^2 = \underline{\quad} + \underline{\quad}$$

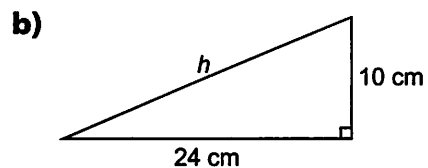
$$h^2 = \underline{\quad} + \underline{\quad}$$

$$h^2 = \underline{\quad}$$

$$h = \sqrt{\underline{\quad}}$$

$$h = \underline{\quad}$$

The length of the hypotenuse is $\underline{\quad}$ cm.



$$h^2 = \underline{\quad} + \underline{\quad}$$

$$h^2 = \underline{\quad} + \underline{\quad}$$

$$h^2 = \underline{\quad}$$

$$h = \sqrt{\underline{\quad}}$$

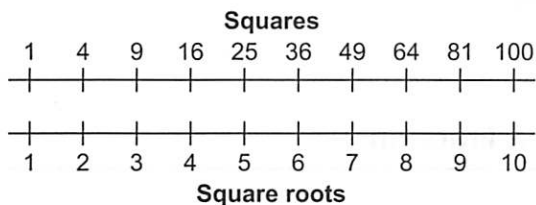
$$h = \underline{\quad}$$

The length of the hypotenuse is $\underline{\quad}$ cm.

1.2 Square Roots of Non-Perfect Squares

FOCUS Approximate the square roots of decimals and fractions that are not perfect squares.

The top number line shows all the perfect squares from 1 to 100.



The bottom number line shows the square root of each number in the top line. You can use these lines to estimate the square roots of fractions and decimals that are not perfect squares.

Example 1 Estimating a Square Root of a Decimal

Estimate: $\sqrt{68.5}$

Solution

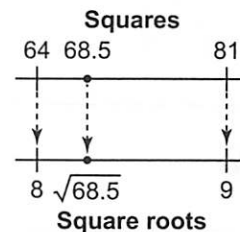
68.5 is between the perfect squares 64 and 81.

So, $\sqrt{68.5}$ is between $\sqrt{64}$ and $\sqrt{81}$.

That is, $\sqrt{68.5}$ is between 8 and 9.

Since 68.5 is closer to 64 than 81, $\sqrt{68.5}$ is closer to 8 than 9.

So, $\sqrt{68.5}$ is between 8 and 9, and closer to 8.



Check

- Estimate each square root.

Explain your estimate.

a) $\sqrt{13.5}$

13.5 is between the perfect squares ___ and ___.

So, $\sqrt{13.5}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$.

That is, $\sqrt{13.5}$ is between ___ and ___.

Since 13.5 is closer to ___ than ___, $\sqrt{13.5}$ is closer to ___ than ___.

So, $\sqrt{13.5}$ is between ___ and ___, and closer to ___.

b) $\sqrt{51.5}$

51.5 is between the perfect squares _____ and _____.

So, $\sqrt{51.5}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$.

That is, $\sqrt{51.5}$ is between _____ and _____.

Since 51.5 is closer to _____ than _____, $\sqrt{51.5}$ is closer to _____ than _____.

So, $\sqrt{51.5}$ is between _____ and _____, and closer to _____.

Example 2 Estimating a Square Root of a Fraction

Estimate: $\sqrt{\frac{3}{10}}$

Solution

Find the closest perfect square to the numerator and denominator.

In the fraction $\frac{3}{10}$:

3 is close to the perfect square 4.

10 is close to the perfect square 9.

$$\text{So, } \sqrt{\frac{3}{10}} \doteq \sqrt{\frac{4}{9}} \text{ and } \sqrt{\frac{4}{9}} = \frac{2}{3}$$

$$\text{So, } \sqrt{\frac{3}{10}} \doteq \frac{2}{3}$$

Check

1. Estimate each square root.

a) $\sqrt{\frac{23}{80}}$

23 is close to the perfect square _____.

80 is close to the perfect square _____.

$$\text{So, } \sqrt{\frac{23}{80}} \doteq \sqrt{\frac{\quad}{\quad}}$$

$$\sqrt{\frac{\quad}{\quad}} = \frac{\quad}{\quad}$$

$$\text{So, } \sqrt{\frac{23}{80}} \doteq \frac{\quad}{\quad}$$

b) $\sqrt{\frac{8}{17}}$

8 is close to the perfect square _____.

17 is close to the perfect square _____.

$$\text{So, } \sqrt{\frac{8}{17}} \doteq \sqrt{\frac{\quad}{\quad}}$$

$$\sqrt{\frac{\quad}{\quad}} = \frac{\quad}{\quad}$$

$$\text{So, } \sqrt{\frac{8}{17}} \doteq \frac{\quad}{\quad}$$

Example 3**Finding a Number with a Square Root between Two Given Numbers**

Identify a decimal that has a square root between 5 and 6.

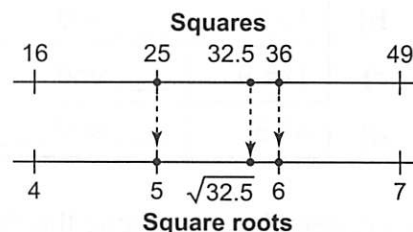
Solution

$5^2 = 25$, so 5 is a square root of 25.

$6^2 = 36$, so 6 is a square root of 36.

So, any decimal between 25 and 36 has a square root between 5 and 6.

Choose 32.5.



Check the answer by using a calculator.

$\sqrt{32.5} \doteq 5.7$, which is between 5 and 6.

So, the decimal 32.5 is one correct answer.

There are many more correct answers.

Check

- 1. a)** Identify a decimal that has a square root between 7 and 8.

Check the answer.

$7^2 = \underline{\quad}$ and $8^2 = \underline{\quad}$

So, any decimal between $\underline{\quad}$ and $\underline{\quad}$ has a square root between 7 and 8.

Choose $\underline{\quad}$.

Check the answer on a calculator.

$\sqrt{\underline{\quad}} \doteq \underline{\quad}$

The decimal $\underline{\quad}$ is one correct answer.

- b)** Identify a decimal that has a square root between 11 and 12.

$\underline{\quad} = \underline{\quad}$ and $\underline{\quad} = \underline{\quad}$

So, any decimal between $\underline{\quad}$ and $\underline{\quad}$ has a square root between 11 and 12.

Choose $\underline{\quad}$.

$\sqrt{\underline{\quad}} \doteq \underline{\quad}$

So, $\underline{\quad}$ is one correct answer.

Practice

1. For each number, name the 2 closest perfect squares and their square roots.

	Number	Two closest perfect squares	Their square roots
a)	44.4	___ and ___	___ and ___
b)	10.8	___ and ___	___ and ___
c)	125.9	___ and ___	___ and ___
d)	87.5	___ and ___	___ and ___

2. For each fraction, name the closest perfect square and its square root for the numerator and for the denominator.

	Fraction	Closest perfect squares	Their square roots
a)	$\frac{5}{11}$	Numerator: ___; denominator: ___	___ and ___
b)	$\frac{17}{45}$	Numerator: ___; denominator: ___	___ and ___
c)	$\frac{3}{24}$	Numerator: ___; denominator: ___	___ and ___
d)	$\frac{11}{62}$	Numerator: ___; denominator: ___	___ and ___

3. Estimate each square root.

Explain.

a) $\sqrt{1.6}$

1.6 is between ___ and ___.

So, $\sqrt{1.6}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$.

That is, $\sqrt{1.6}$ is between ___ and ___.

Since 1.6 is closer to ___ than ___, $\sqrt{1.6}$ is closer to ___ than ___.

So, $\sqrt{1.6}$ is between ___ and ___, and closer to ___.

b) $\sqrt{44.5}$

44.5 is between ___ and ___.

So, $\sqrt{44.5}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$.

That is, $\sqrt{44.5}$ is between ___ and ___.

Since 44.5 is closer to ___ than ___, $\sqrt{44.5}$ is closer to ___ than ___.

So, $\sqrt{44.5}$ is between ___ and ___, and closer to ___.

c) $\sqrt{75.8}$

75.8 is between ____ and ____.

So, $\sqrt{75.8}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$.

That is, $\sqrt{75.8}$ is between ____ and ____.

Since 75.8 is closer to ____ than ____, $\sqrt{75.8}$ is closer to ____ than ____.

So, $\sqrt{75.8}$ is between ____ and ____, and closer to ____.

4. Estimate each square root. Explain.

a) $\sqrt{\frac{7}{15}}$

7 is close to ____; 15 is close to ____.

So, $\sqrt{\frac{7}{15}} \doteq \sqrt{\frac{\quad}{\quad}}$

$\doteq \frac{\quad}{\quad}$

b) $\sqrt{\frac{2}{7}}$

2 is close to ____; 7 is close to ____.

So, $\sqrt{\frac{2}{7}} \doteq \sqrt{\frac{\quad}{\quad}}$

$\doteq \frac{\quad}{\quad}$

c) $\sqrt{\frac{35}{37}}$

35 is close to ____; 37 is close to ____.

So, $\sqrt{\frac{35}{37}} \doteq \sqrt{\frac{\quad}{\quad}}$

$\doteq \frac{\quad}{\quad}$

d) $\sqrt{\frac{99}{122}}$

99 is close to ____; 122 is close to ____.

So, $\sqrt{\frac{99}{122}} \doteq \sqrt{\frac{\quad}{\quad}}$

$\doteq \frac{\quad}{\quad}$

5. Identify a decimal that has a square root between the two given numbers.

Check the answer.

a) 1 and 2

$1^2 = \quad$ and $2^2 = \quad$

So, any number between ____ and ____ has a square root between 1 and 2.

Choose ____.

Check: $\sqrt{\quad} \doteq \quad$

The decimal ____ is one possible answer.

b) 8 and 9

$8^2 = \quad$ and $9^2 = \quad$

So, any number between ____ and ____ has a square root between 8 and 9.

Choose ____.

Check: $\sqrt{\quad} \doteq \quad$

The decimal ____ is one possible answer.

c) 2.5 and 3.5

_____ = _____ and _____ = _____

So, any number between _____ and _____ has a square root between 2.5 and 3.5.

Choose _____.

Check: $\sqrt{\text{_____}} \doteq \text{_____}$

The decimal _____ is one correct answer.

d) 20 and 21

_____ = _____ and _____ = _____

So, any number between _____ and _____ has a square root between 20 and 21.

Choose _____.

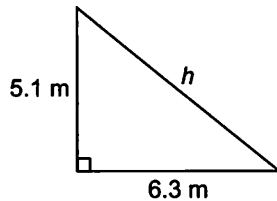
Check: $\sqrt{\text{_____}} \doteq \text{_____}$

The decimal _____ is one correct answer.

6. Determine the length of the hypotenuse in each right triangle.

Write each answer to the nearest tenth.

a)



$$h^2 = 5.1^2 + 6.3^2$$

$$h^2 = \text{_____} + \text{_____}$$

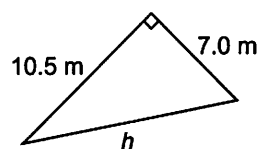
$$h^2 = \text{_____}$$

$$h = \sqrt{\text{_____}}$$

$$h \doteq \text{_____}$$

So, h is about _____ m.

b)



$$h^2 = \text{_____} + \text{_____}$$

$$h^2 = \text{_____} + \text{_____}$$

$$h^2 = \text{_____}$$

$$h = \sqrt{\text{_____}}$$

$$h \doteq \text{_____}$$

So, h is about _____ m.

CHECKPOINT

Can you ...

- Identify decimals and fractions that are perfect squares?
- Find the square roots of decimals and fractions that are perfect squares?
- Approximate the square roots of decimals and fractions that are not perfect squares?

1.1 1. Calculate the number whose square root is:

a) $\frac{2}{7}$

$\frac{2}{7} \times \frac{2}{7} = \underline{\hspace{2cm}}$

$\frac{2}{7}$ is a square root of $\underline{\hspace{2cm}}$.

b) $\frac{8}{11}$

$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$

$\frac{8}{11}$ is a square root of $\underline{\hspace{2cm}}$.

c) 0.1

$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$

0.1 is a square root of $\underline{\hspace{2cm}}$.

d) 1.4

$1.4 \times 1.4 = \underline{\hspace{2cm}}$

1.4 is a square root of $\underline{\hspace{2cm}}$.

2. Identify the fractions that are perfect squares.

The first one has been done for you.

	Fraction	Is numerator a perfect square?	Is denominator a perfect square?	Is fraction a perfect square?
a)	$\frac{64}{75}$	Yes; $8 \times 8 = 64$	No	No
b)	$\frac{9}{25}$	$\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$
c)	$\frac{25}{55}$	$\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$

3. Find each square root.

a) $\sqrt{\frac{9}{49}} = \sqrt{\frac{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}}$
 $= \underline{\hspace{2cm}}$

b) $\sqrt{\frac{16}{25}} = \sqrt{\frac{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}}$
 $= \underline{\hspace{2cm}}$

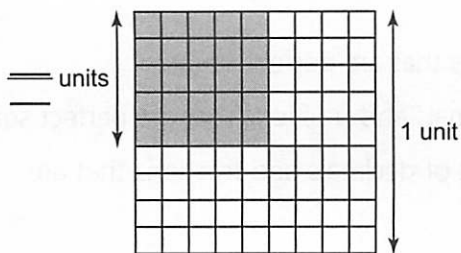
c) $\sqrt{\frac{36}{121}} = \sqrt{\frac{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}{\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}}}$
 $= \underline{\hspace{2cm}}$

4. a) Put a check mark beside each decimal that is a perfect square.

- i) 4.84 ii) 3.63 iii) 98.01 iv) 67.24

b) Explain how you identified the perfect squares in part a.

5. a) Find the area of the shaded square.



$$\text{Area} = (\text{Length})^2$$

$$= (\quad)^2$$

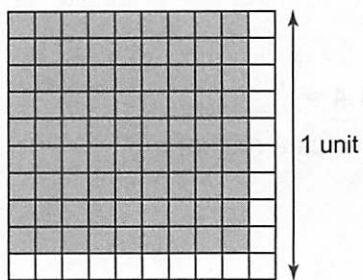
$$= \quad \times \quad$$

$$= \quad$$

The area is _____ square units.

b) Find the side length of the shaded square.

$$\text{Area} = \frac{81}{100} \text{ square units}$$



$$\text{Length} = \sqrt{\text{Area}}$$

$$= \sqrt{\quad}$$

$$= \sqrt{\quad \times \quad}$$

$$= \quad$$

The side length is _____ units.

1.2 6. Estimate each square root.

Explain.

a) $\sqrt{7.5}$

7.5 is between _____ and _____.

So, $\sqrt{7.5}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$.

That is, $\sqrt{7.5}$ is between _____ and _____.

Since 7.5 is closer to _____ than _____, $\sqrt{7.5}$ is closer to _____ than _____.

So, $\sqrt{7.5}$ is between _____ and _____, and closer to _____.

b) $\sqrt{66.6}$

66.6 is between _____ and _____.

So, $\sqrt{66.6}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$.

That is, $\sqrt{66.6}$ is between _____ and _____.

Since 66.6 is closer to _____ than _____, $\sqrt{66.6}$ is closer to _____ than _____.

So, $\sqrt{66.6}$ is between _____ and _____, and closer to _____.

7. Estimate each square root.

a) $\sqrt{\frac{15}{79}}$

15 is close to ____; 79 is close to ____.

So, $\sqrt{\frac{15}{79}} \doteq \sqrt{\frac{\underline{\quad}}{\underline{\quad}}}$
 $\doteq \frac{\underline{\quad}}{\underline{\quad}}$

b) $\sqrt{\frac{23}{50}}$

23 is close to ____; 50 is close to ____.

So, $\sqrt{\frac{23}{50}} \doteq \sqrt{\frac{\underline{\quad}}{\underline{\quad}}}$
 $\doteq \frac{\underline{\quad}}{\underline{\quad}}$

8. Identify a decimal whose square root is between the given numbers.

Check your answer.

a) 2 and 3

$2^2 = \underline{\quad}$ and $3^2 = \underline{\quad}$

So, any number between ____ and ____ has a square root between 2 and 3.

Choose ____.

Check: $\sqrt{\underline{\quad}} \doteq \underline{\quad}$

The decimal ____ is one correct answer.

b) 6 and 7

$6^2 = \underline{\quad}$ and $7^2 = \underline{\quad}$

So, any number between ____ and ____ has a square root between 6 and 7.

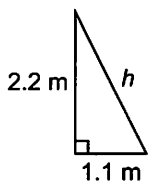
Choose ____.

$\sqrt{\underline{\quad}} \doteq \underline{\quad}$

The decimal ____ is one correct answer.

9. Find the length of each hypotenuse.

a)



$h^2 = \underline{\quad} + \underline{\quad}$

$h^2 = \underline{\quad} + \underline{\quad}$

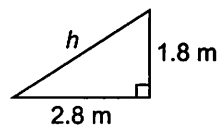
$h^2 = \underline{\quad}$

$h = \sqrt{\underline{\quad}}$

$h \doteq \underline{\quad}$

The length of the hypotenuse is about ____ m.

b)



$h^2 = \underline{\quad} + \underline{\quad}$

$h^2 = \underline{\quad} + \underline{\quad}$

$h^2 = \underline{\quad}$

$h = \sqrt{\underline{\quad}}$

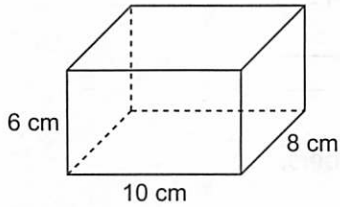
$h \doteq \underline{\quad}$

The length of the hypotenuse is about ____ m.

1.3 Skill Builder

Surface Areas of Rectangular Prisms

The **surface area** of a rectangular prism is the sum of the areas of its 6 rectangular faces. Look for matching faces with the same areas.



The matching faces in each pair have the same area. We find the area of one face and multiply by 2.

For each rectangular face, area equals its length times its width.

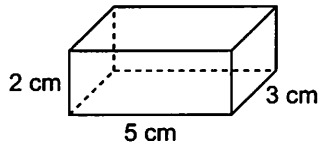
Matching Faces	Diagram	Corresponding Area (cm ²)
		$2(10 \times 6) = 120$
		$2(10 \times 8) = 160$
		$2(8 \times 6) = 96$
Total		376

The surface area is 376 cm².

Check

1. Determine the surface area of each rectangular prism.

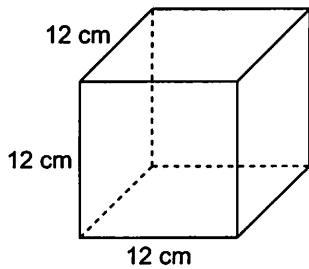
a)



Matching Faces	Diagram	Corresponding Area (cm ²)
Front Back		$2(__ \times __) = __$
Top Bottom		$2(__ \times __) = __$
Right Left		$2(__ \times __) = __$
Total		$__$

The surface area is $__ \text{ cm}^2$.

b)



Matching Faces	Diagram	Corresponding Area (cm ²)
Front Back		$2(__ \times __) = __$
Top Bottom		$2(__ \times __) = __$
Right Left		$2(__ \times __) = __$
Total		$__$

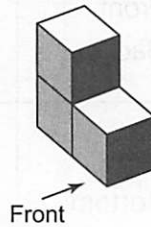
The surface area is $__ \text{ cm}^2$.

1.3 Surface Areas of Objects Made from Right Rectangular Prisms

FOCUS Find the surface areas of objects made from rectangular prisms.

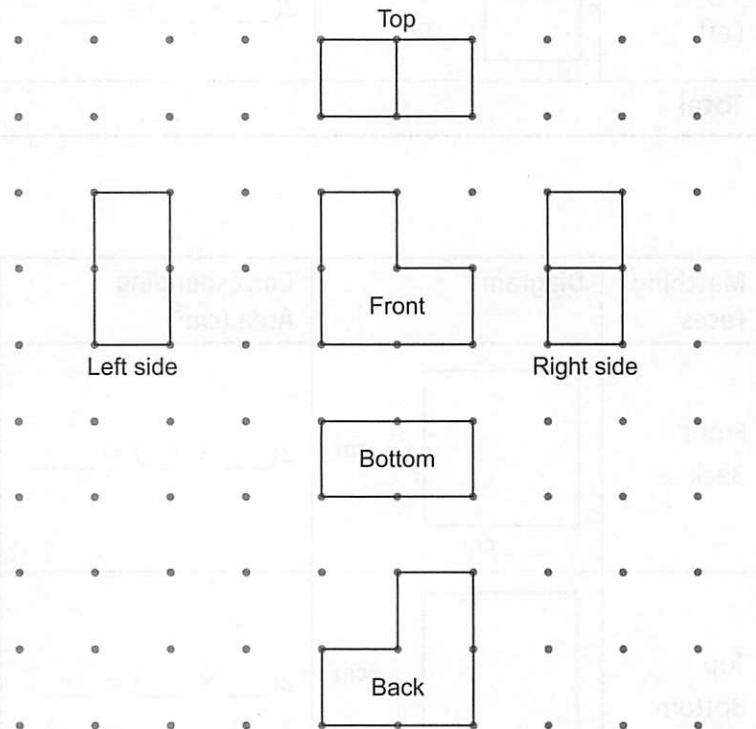
Example 1 Finding the Surface Area of an Object Made from Cubes

Make this object with 1-cm cubes.
What is the surface area of the object?



Solution

Think of tracing each face, or "opening" the object.



Turn the object to see each view.

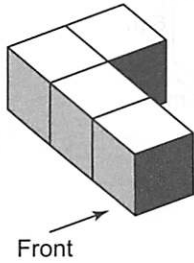
Look for matching views.

Matching Views	Corresponding Area (cm ²)
Front / Back	$2(3) = 6$
Top / Bottom	$2(2) = 4$
Right / Left	$2(2) = 4$
Total	14

The surface area is 14 cm².

Check

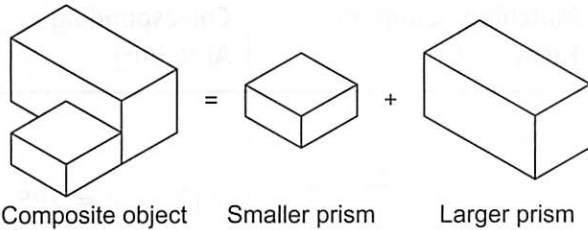
1. Make this object with 1-cm cubes, then find its surface area.



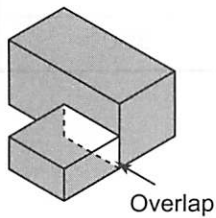
Matching Views	Diagram	Corresponding Area (cm ²)
Front Back		$2(\underline{\quad}) = \underline{\quad}$
Top Bottom		$2(\underline{\quad}) = \underline{\quad}$
Right Left		$2(\underline{\quad}) = \underline{\quad}$
Total		$\underline{\quad}$

The surface area is $\underline{\quad}$ cm².

A **composite object** is made from 2 or more objects.



To find the surface area of a composite object, imagine dipping the object in paint. The surface area is the area of all the faces covered in paint.

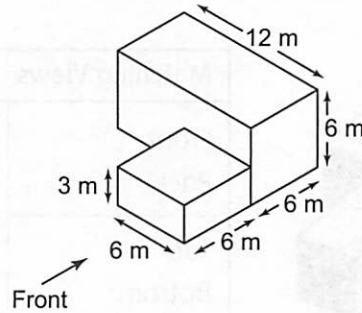


*Where objects overlap,
there is a hidden surface.
The paint doesn't reach
the hidden surface.*

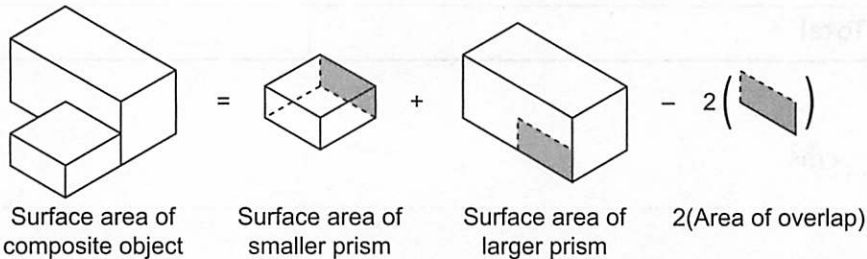
The overlap is not painted, so it is not part of the surface area.

Example 2 Finding the Surface Area of a Composite Object

Find the surface area of this composite object.



Solution



Surface area of smaller prism

Matching Faces	Diagram	Corresponding Area (m^2)
Front Back Right Left		$4(6 \times 3) = 72$
Top Bottom		$2(6 \times 6) = 72$
Total		144

The surface area is $144 m^2$.

Surface area of larger prism

Matching Faces	Diagram	Corresponding Area (m^2)
Front Back Top Bottom		$4(12 \times 6) = 288$
Right Left		$2(6 \times 6) = 72$
Total		360

The surface area is $360 m^2$.

Area of overlap

Diagram	Corresponding Area (m^2)
	$6 \times 3 = 18$

The area of overlap is $18 m^2$.

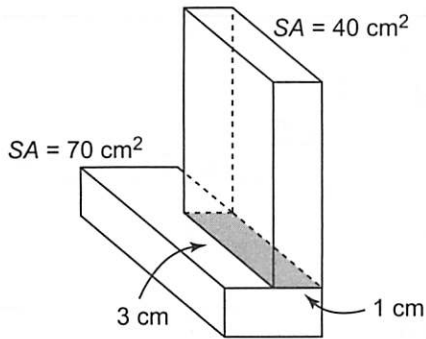
SA of composite object = $144 + 360 - 2(18) = 468$

The surface area of the composite object is $468 m^2$.

SA means surface area.

Check

1. The diagram shows the surface areas of the two prisms that make up a composite object.



- a) What is the area of the overlap?

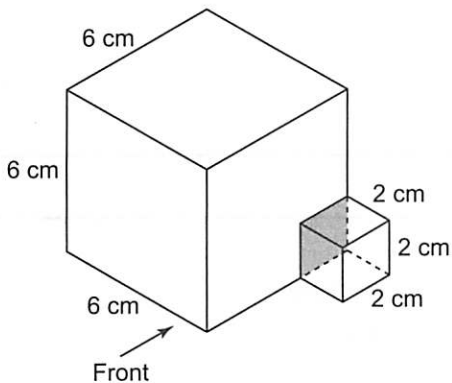
The overlap is a ____-cm by ____-cm rectangle.

$$\begin{aligned} \text{Area of overlap} &= \text{____ cm} \times \text{____ cm} \\ &= \text{____ cm}^2 \end{aligned}$$

- b) What is the surface area of the composite object?

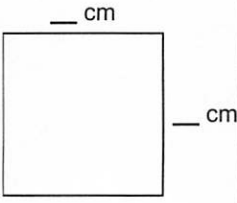
$$\begin{aligned} \text{SA composite object} &= \text{SA smaller prism} + \text{SA larger prism} - 2(\text{Area of overlap}) \\ &= \text{____ cm}^2 + \text{____ cm}^2 - 2(\text{____}) \text{ cm}^2 \\ &= \text{____ cm}^2 \end{aligned}$$

2. Find the surface area of this composite object.



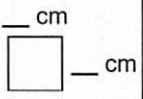
A cube has ____ congruent faces.

Surface area of larger cube

Matching Faces	Diagram	Corresponding Area (cm ²)
Front Back Top Bottom Right Left		6(___ × ___) = ____
Total		_____

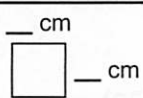
The surface area is ____ cm².

Surface area of smaller cube

Matching Faces	Diagram	Corresponding Area (cm ²)
Front Back Top Bottom Right Left		6(___ × ___) = ____
Total		_____

The surface area is ____ cm².

Area of overlap

Diagram	Corresponding Area (cm ²)
	___ × ___ = ___

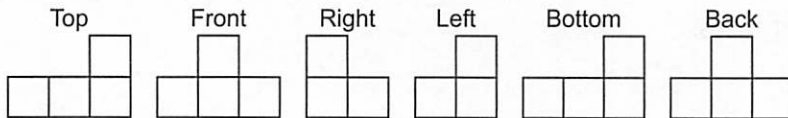
The area of overlap is ____ cm².

$$\begin{aligned}
 \text{SA composite object} &= \text{SA larger cube} + \text{SA smaller cube} - 2(\text{Area of overlap}) \\
 &= \text{_____} + \text{_____} - 2(\text{___}) \\
 &= \text{_____}
 \end{aligned}$$

The surface area of the composite object is ____ cm².

Practice

- The diagram shows the 6 views of an object made from 1-cm cubes. Identify pairs of matching views in the first column of the table. Then, find the surface area of the object.

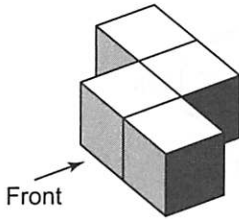


Matching Views	Corresponding Area (cm ²)
Front / _____	_____
Top / _____	_____
Right / _____	_____
Total	_____

The surface area is ____ cm².

2. Each object is made with 1-cm cubes. Find the surface area of each object.

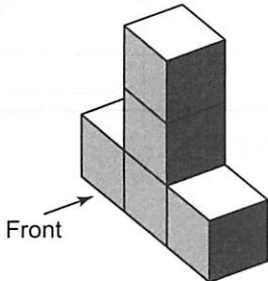
a)



Matching Views	Diagram	Corresponding Area (cm ²)
Front Back		2() = ____
Top Bottom		2() = ____
Right Left		2() = ____
Total		____

The surface area is ____ cm².

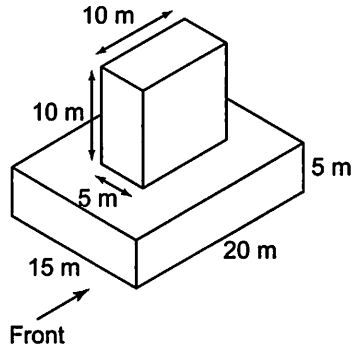
b)



Matching Views	Diagram	Corresponding Area (cm ²)
Front Back		2() = ____
Top Bottom		_____
Right Left		_____
Total		____

The surface area is ____ cm².

3. Find the surface area of this composite object.



Surface area of larger prism

Matching Faces	Diagram	Corresponding Area (m ²)
Front Back		$2(__ \times __) = __$
Top Bottom		_____
Right Left		_____
Total		_____

The surface area is ___ m².

Surface area of smaller prism

Matching Faces	Diagram	Corresponding Area (m ²)
Front Back		$2(__ \times __) = __$
Top Bottom		_____
Right Left		_____
Total		_____

The surface area is ___ m².

Area of overlap

Diagram	Corresponding Area (m ²)
	$__ \times __ = __$

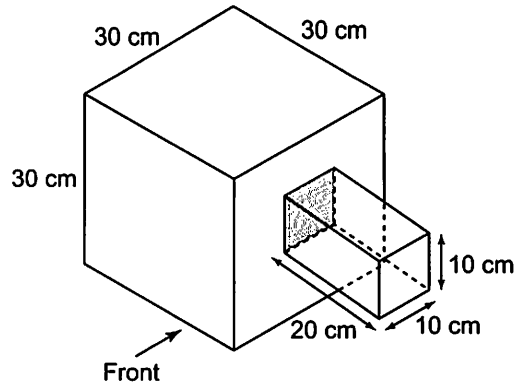
The area of overlap is ___ m².

Surface area of composite object

$$\begin{aligned}
 \text{SA composite object} &= \text{_____} + \text{_____} - \text{_____} \\
 &= __ + __ - 2(__) \\
 &= __
 \end{aligned}$$

The surface area of the composite object is ___ m².

4. Find the surface area of this composite object.



Surface area of cube

Matching Faces	Diagram	Corresponding Area (cm ²)
Front / Back Top / Bottom Right / Left		$6(__ \times __) = __$
Total		$__$

The surface area is $__ \text{ cm}^2$.

Surface area of rectangular prism

Matching Faces	Diagram	Corresponding Area (cm ²)
Front / Back		$2(__ \times __) = __$
Top / Bottom		$__$
Right / Left		$__$
Total		$__$

The surface area is $__ \text{ cm}^2$.

Area of overlap

Diagram	Corresponding Area (cm ²)
	$__ \times __ = __$

The area of overlap is $__ \text{ cm}^2$.

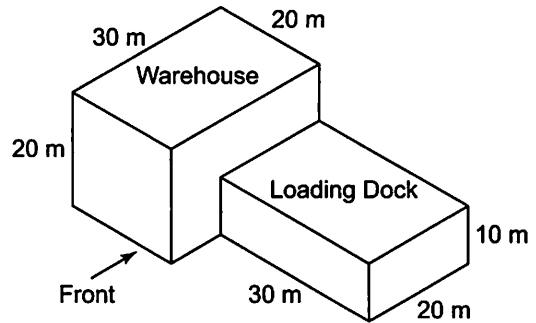
Surface area of composite object

$$\begin{aligned}
 \text{SA composite object} &= __ + __ - __ \\
 &= __ + __ - __ \\
 &= __
 \end{aligned}$$

The surface area of the composite object is $__ \text{ cm}^2$.

5. A loading dock is attached to one wall of a warehouse. The exterior of the buildings is to be painted at a cost of \$2.50/m². How much will it cost to paint the buildings?

Will the bottom of the warehouse and loading dock be painted? _____



Surface area of warehouse to be painted

Matching Faces	Diagram	Corresponding Area (m ²)
Front Back		2(____ × ____) = ____
Top Sides		3(____ × ____) = ____
Total		_____

The surface area of the warehouse to be painted is ____ m².

Area of overlap

Diagram	Corresponding Area (m ²)
	____ × ____ = ____

The area of overlap is ____ m².

Surface area of composite object to be painted

____ + ____ - ____ = ____

The surface area of the composite object to be painted is ____ m².

So, the area to be painted is ____ m².

The cost per square metre is: \$ ____

The cost to paint the buildings is: ____ × \$ ____ = ____

Surface area of loading dock to be painted

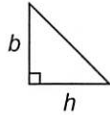
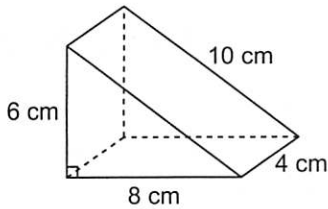
Matching Faces	Diagram	Corresponding Area (m ²)
Front Back		2(____ × ____) = ____
Top		____ × ____ = ____
Sides		2(____ × ____) = ____
Total		_____

The surface area of the loading dock to be painted is ____ m².

1.4 Skill Builder

Surface Areas of Triangular Prisms

To find the surface area of a right triangular prism, add the areas of its 5 faces.
Look for matching faces with the same areas.



$$A = \frac{1}{2}bh$$

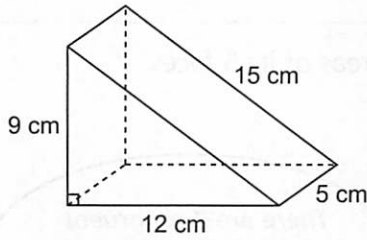
There are 2 congruent triangular faces. Find the area of one, then multiply it by 2.

Matching Faces	Diagram	Corresponding Area (cm ²)
Triangular		$2\left(\frac{1}{2} \times 6 \times 8\right) = 48$
Rectangular		$10 \times 4 = 40$
		$6 \times 4 = 24$
		$8 \times 4 = 32$
Total		144

The surface area is 144 cm².

Check

1. Find the surface area of the triangular prism.



Matching Faces	Diagram	Corresponding Area (cm ²)
Triangular		$2\left(\frac{1}{2} \times \underline{\quad} \times \underline{\quad}\right) = \underline{\quad}$
Rectangular		$\underline{\quad} \times \underline{\quad} = \underline{\quad}$
		$\underline{\quad} \times \underline{\quad} = \underline{\quad}$
		$\underline{\quad} \times \underline{\quad} = \underline{\quad}$
Total		$\underline{\quad}$

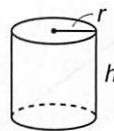
The surface area is $\underline{\quad}$ cm².

Surface Areas of Cylinders

To find the surface area of a right cylinder, add the areas of:

- the 2 circular faces
- the curved surface

Look for matching faces with the same areas.

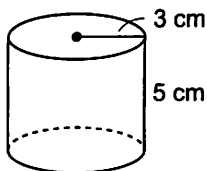


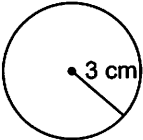
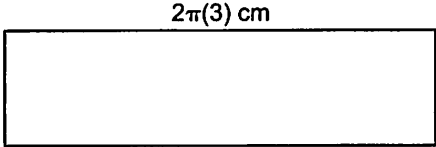
Matching Faces	Diagram	Corresponding Area
Top Bottom		$2 \times \pi r^2$
Curved surface		$2\pi rh$

The side can be unrolled into a rectangle, whose length is the circumference of the circle.

The surface area is: $2\pi r^2 + 2\pi rh$

To calculate the surface area of this cylinder:



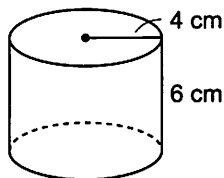
Matching Faces	Diagram	Corresponding Area (cm ²)
Top Bottom		$2 \times \pi \times 3^2$ $\doteq 56.55$
Curved surface		$2 \times \pi \times 3 \times 5$ $\doteq 94.25$
Total		150.80

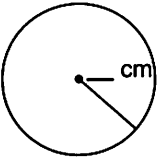
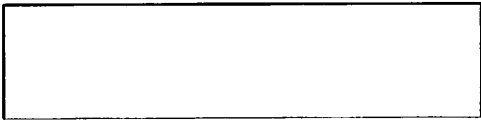
The dimensions of the cylinder are given to the nearest centimetre, so we give the surface area to the nearest square centimetre.

The surface area is about 151 cm².

Check

1. Find the surface area of the cylinder.



Matching Faces	Diagram	Corresponding Area (cm ²)
Top Bottom		$__ \times __ \times __$ $\doteq ______$
Curved surface		$__ \times __ \times __ \times __$ $\doteq ______$
Total		$______$

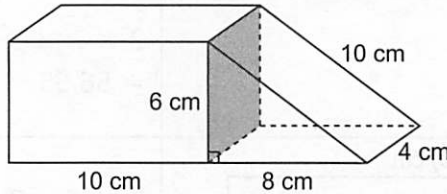
The surface area is about $______ \text{ cm}^2$.

1.4 Surface Areas of Other Composite Objects

FOCUS Find the surface areas of composite objects made from right prisms and right cylinders.

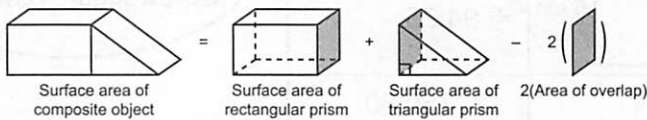
Example 1 Finding the Surface Area of a Composite Object Made from a Rectangular Prism and a Triangular Prism

Find the surface area of this composite object.



The shaded area is the area of overlap.

Solution



Surface area of rectangular prism

Matching Faces	Diagram	Corresponding Area (cm ²)
Front Back	10 cm, 6 cm	$2(6 \times 10) = 120$
Top Bottom	10 cm, 4 cm	$2(10 \times 4) = 80$
Right Left	6 cm, 4 cm	$2(6 \times 4) = 48$
Total		248

The surface area is 248 cm².

Area of overlap

Diagram	Corresponding Area (cm ²)
6 cm, 4 cm	$6 \times 4 = 24$

The area of overlap is 24 cm².

Surface area of composite object = 248 + 144 - 2(24) = 344

The surface area of the composite object is 344 cm².

Surface area of triangular prism

Matching Faces	Diagram	Corresponding Area (cm ²)
Triangular	6 cm, 8 cm, 10 cm	$2\left(\frac{1}{2} \times 6 \times 8\right) = 48$
Rectangular	10 cm, 4 cm	$10 \times 4 = 40$
	6 cm, 4 cm	$6 \times 4 = 24$
	8 cm, 4 cm	$8 \times 4 = 32$
Total		144

The surface area is 144 cm².

Check

1. The diagram shows the surface area of the two prisms that make up a composite object.

a) What is the area of the overlap?

The overlap is a ___-cm by ___-cm rectangle.

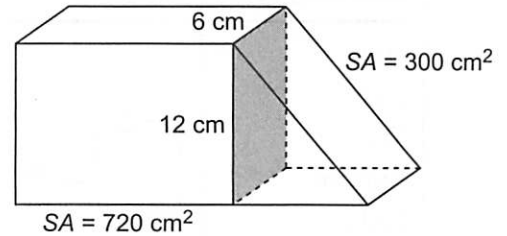
Area of overlap = ___ cm × ___ cm = ___ cm²

b) What is the surface area of the composite object?

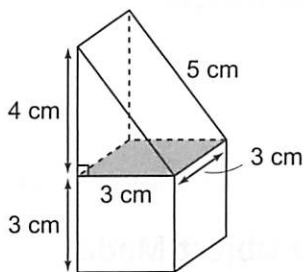
Surface area of composite object = Surface area of 2 prisms – 2(Area of overlap)

$$= \underline{\quad} + \underline{\quad} - \underline{\quad} = \underline{\quad}$$

The surface area of the composite object is _____.



2. Find the surface area of this composite object.

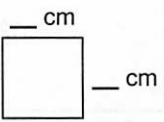


Surface area of triangular prism

Matching Faces	Diagram	Corresponding Area (cm ²)
Triangular		$2(\underline{\quad} \times \underline{\quad} \times \underline{\quad}) = \underline{\quad}$
Rectangular		$\underline{\quad} \times \underline{\quad} = \underline{\quad}$
		$\underline{\quad} \times \underline{\quad} = \underline{\quad}$
		$\underline{\quad} \times \underline{\quad} = \underline{\quad}$
Total		$\underline{\quad}$

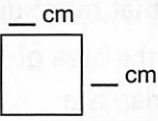
The surface area is ___ cm².

Surface area of cube

Matching Faces	Diagram	Corresponding Area (cm ²)
Front Back Top Bottom Right Left		$6(\text{___} \times \text{___}) = \text{___}$
Total		___

The surface area is ___ cm².

Area of overlap

Diagram	Corresponding Area (cm ²)
	$\text{___} \times \text{___} = \text{___}$

The area of overlap is ___ cm².

Surface area of composite object = Surface area of 2 prisms – 2(Area of overlap)

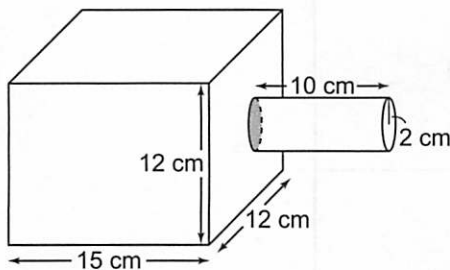
$$= \text{___} + \text{___} - \text{___}$$

$$= \text{___}$$

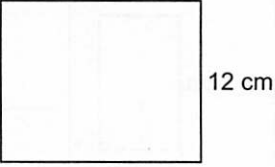
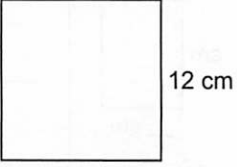
The surface area of the composite object is ___ cm².

Example 2 Finding the Surface Area of a Composite Object Made from a Rectangular Prism and a Cylinder

Find the surface area of this object.

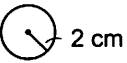
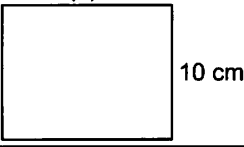


Surface area of rectangular prism

Matching Faces	Diagram	Corresponding Area (cm ²)
Front Back Top Bottom		$4(12 \times 15) = 720$
Right Left		$2(12 \times 12) = 288$
Total		1008

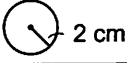
The surface area is 1008 cm².

Surface area of cylinder

Matching Faces	Diagram	Corresponding Area (cm ²)
Top Bottom	 2 cm	$2 \times \pi \times 2^2 \doteq 25.13$
Curved surface	 10 cm	$2 \times \pi \times 2 \times 10 \doteq 125.67$
Total		150.80

The surface area is about 150.80 cm².

Area of overlap

Diagram	Corresponding Area (cm ²)
 2 cm	$\pi \times 2^2 \doteq 12.57$

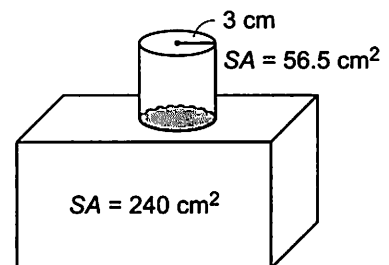
The area of overlap is about 12.57 cm².

$$\begin{aligned} \text{SA composite object} &= \text{SA rectangular prism} + \text{SA cylinder} - 2(\text{Area of overlap}) \\ &\doteq 1008 + 150.80 - 2(12.57) \\ &\doteq 1133.66 \end{aligned}$$

The surface area is about 1134 cm².

Check

1. The diagram shows the surface area of the rectangular prism and cylinder that make up a composite object.



- a) What is the area of the overlap?

The overlap is a _____.

Area of overlap = _____

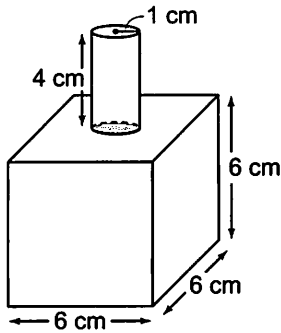
\doteq _____ cm²

- b) What is the surface area of the composite object?

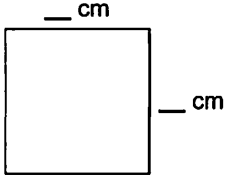
$$\begin{aligned} \text{SA composite object} &= \text{SA} \text{ _____} + \text{SA} \text{ _____} - 2(\text{_____}) \\ &= \text{_____} + \text{_____} - \text{_____} \\ &= \text{_____} \end{aligned}$$

The surface area of the composite object is about _____ cm².

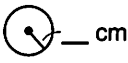

2. Find the surface area of this composite object.



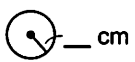
Surface area of cube

Matching Faces	Diagram	Corresponding Area (cm ²)
Front Back Top Bottom Right Left		$6(\underline{\quad} \times \underline{\quad}) = \underline{\quad}$
Total		$\underline{\quad}$

Surface area of cylinder

Matching Faces	Diagram	Corresponding Area (cm ²)
Top Bottom		$\underline{\quad} \times \underline{\quad} \times \underline{\quad} \doteq \underline{\quad}$
Curved surface		$\underline{\quad} \times \underline{\quad} \times \underline{\quad} \times \underline{\quad} \doteq \underline{\quad}$
Total		$\underline{\quad}$

Area of overlap

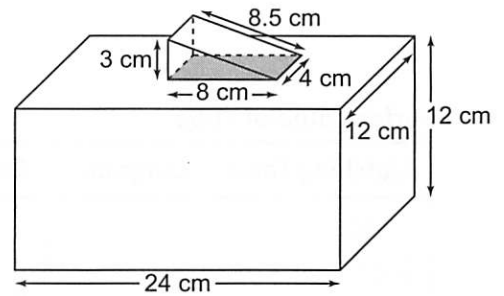
Diagram	Corresponding Area (cm ²)
	$\underline{\quad} \times \underline{\quad} \doteq \underline{\quad}$

$$\begin{aligned}
 \text{SA composite object} &= \text{SA } \underline{\quad} + \text{SA } \underline{\quad} - 2(\underline{\quad}) \\
 &\doteq \underline{\quad} + \underline{\quad} - \underline{\quad} \\
 &\doteq \underline{\quad}
 \end{aligned}$$

The surface area of the composite object is about $\underline{\quad}$ cm².

Practice

1. Find the surface area of this composite object.



Surface area of rectangular prism

Matching Faces	Diagram	Corresponding Area (cm ²)
Front Back Top Bottom		_____ = _____
Right Left		_____ = _____
Total		_____

The surface area is _____ cm².

Surface area of triangular prism

Matching Faces	Diagram	Corresponding Area (cm ²)
Triangular		_____
Rectangular		_____

Total		_____

The surface area is _____ cm².

Area of overlap

Diagram	Area (cm ²)
	___ × ___ = ___

The area of overlap is _____ cm².

Surface area of composite object

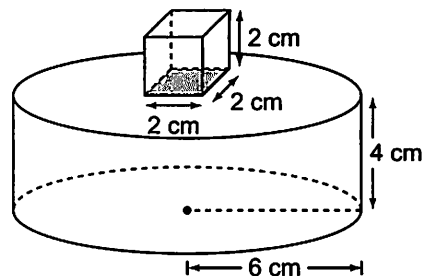
SA composite object

= _____

= _____

The surface area of the composite object is _____ cm².

2. Find the surface area of this composite object.



Surface area of cube

Matching Faces	Diagram	Corresponding Area (cm ²)
Front Back Top Bottom		$6(\text{---} \times \text{---}) = \text{---}$
Total		---

The surface area is --- cm².

Surface area of cylinder

Matching Faces	Diagram	Corresponding Area (cm ²)
Top Bottom		$\text{---} \times \text{---} \times \text{---} \div \text{---}$
Curved surface		$\text{---} \times \text{---} \times \text{---} \times \text{---} \div \text{---}$
Total		---

The surface area is about --- cm².

Area of overlap

Diagram	Corresponding Area (cm ²)
	$\text{---} \times \text{---} = \text{---}$

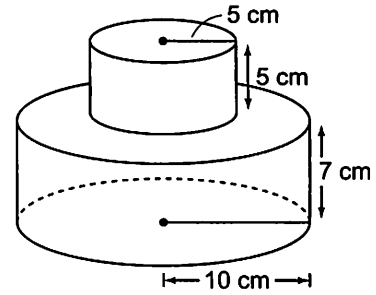
The area of overlap is --- cm².

Surface area of composite object

SA composite object \div --- + --- - ---
 \div ---

The surface area of the composite object is about --- cm².

3. Calculate the surface area of the cake at the right.
Write your answer to the nearest tenth.



Surface area of smaller cake

Matching Faces	Diagram	Corresponding Area (cm ²)
Top Bottom		___ × ___ × ___ ÷ _____
Curved surface		___ × ___ × ___ × ___ ÷ _____
Total		_____

The surface area is about _____ cm².

Surface area of larger cake

Matching Faces	Diagram	Corresponding Area (cm ²)
Top Bottom		___ × ___ × ___ ÷ _____
Curved surface		___ × ___ × ___ × ___ ÷ _____
Total		_____

The surface area is about _____ cm².

Area of overlap

Diagram	Corresponding Area (cm ²)
	___ × ___ ÷ _____

The area of overlap is about _____ cm².

Surface area of cake ÷ _____ + _____ - _____
 ÷ _____

The surface area of the cake is about _____ cm².

Unit 1 Puzzle

Square and Square-Root Days

A date in a given year can be written as the month number followed by the day number. For example, October 25 can be written as 10/25.

- In a *square-root day*, the month is the square root of the day. For example, March 9 is a square-root day because it is written as 3/9, and 3 is the square root of 9.

List all the square-root days in a year.

- In a *square day*, the month is the square of the day. For example, April 2 is a square day because it is written as 4/2, and 4 is the square of 2.

List all the square days in a year.

- A *square year* is a year which is a perfect square. For example, the year 1600 is a square year because $1600 = 40 \times 40$.

List all the square years from 1000 to the present.

Unit 1 Study Guide

Skill	Description	Example
Identify fractions that are perfect squares and find their square roots.	A fraction is a perfect square if it can be written as the product of 2 equal fractions. The square root is one of the 2 equal fractions.	$\frac{16}{25} = \frac{4}{5} \times \frac{4}{5}$ $\sqrt{\frac{16}{25}} = \frac{4}{5}$
Identify decimals that are perfect squares.	Use a calculator. The square root is a repeating or terminating decimal.	$\sqrt{1.69} = 1.3$
Estimate square roots of numbers that are not perfect squares.	Find perfect squares close to the number. Use the squares and square roots number lines.	$\sqrt{10} \approx \sqrt{\frac{4}{9}} \approx \frac{2}{3}$ <p>3 is close to 4; 10 is close to 9.</p> <p>Squares</p> <p>Square roots</p>
Calculate the surface area of a composite object.	Add the areas of each of the 6 views. Or Add surface areas of the parts, then subtract for the overlap.	<p>Front</p> <p>The surface area is 14 square units.</p> <p>Top Front Right side Bottom Left side Back</p> <p>SA = 125.66 cm² Area = 12.57 cm² SA = 216 cm²</p> $SA = 216 + 125.66 - 2(12.57)$ $= 316.52$ <p>The surface area is about 317 cm².</p>

Unit 1 Review

1.1 1. Calculate the number whose square root is:

a) $\frac{3}{7}$

$\frac{3}{7} \times \frac{3}{7} = \frac{\quad}{\quad}$

$\frac{3}{7}$ is a square root of $\frac{\quad}{\quad}$.

b) 9.9

$9.9 \times 9.9 = \underline{\hspace{2cm}}$

9.9 is a square root of $\underline{\hspace{2cm}}$.

2. Complete the table.

	Fraction	Is numerator a perfect square?	Is denominator a perfect square?	Is fraction a perfect square?
a)	$\frac{25}{81}$	$\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$
b)	$\frac{7}{4}$	$\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$
c)	$\frac{49}{65}$	$\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$

3. Complete the table.

	Decimal	Value of Square Root	Type of Decimal	Is decimal a perfect square?
a)	5.29	$\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$ $\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$
b)	156.25	$\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$ $\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$
c)	6.4	$\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$ $\underline{\hspace{2cm}}$	$\underline{\hspace{2cm}}$

4. Find the square root of each number.

a) $\sqrt{\frac{25}{81}} = \underline{\hspace{2cm}}$

b) $\sqrt{59.29} = \underline{\hspace{2cm}}$

1.2 5. Estimate $\sqrt{14.5}$. Explain your estimate.

14.5 is between $\underline{\hspace{1cm}}$ and $\underline{\hspace{1cm}}$.

So, $\sqrt{14.5}$ is between $\sqrt{\underline{\hspace{1cm}}}$ and $\sqrt{\underline{\hspace{1cm}}}$. That is, $\sqrt{14.5}$ is between $\underline{\hspace{1cm}}$ and $\underline{\hspace{1cm}}$.

Since 14.5 is closer to $\underline{\hspace{1cm}}$ than $\underline{\hspace{1cm}}$, $\sqrt{14.5}$ is closer to $\underline{\hspace{1cm}}$ than $\underline{\hspace{1cm}}$.

So, $\sqrt{14.5}$ is between $\underline{\hspace{1cm}}$ and $\underline{\hspace{1cm}}$, and closer to $\underline{\hspace{1cm}}$.

6. Estimate each square root. Explain.

a) $\sqrt{\frac{2}{13}}$

2 is close to ____; 13 is close to ____.

So, $\sqrt{\frac{2}{13}} \doteq \sqrt{\frac{\underline{\quad}}{\underline{\quad}}}$
 $\doteq \frac{\underline{\quad}}{\underline{\quad}}$

b) $\sqrt{\frac{11}{70}}$

11 is close to ____; 70 is close to ____.

So, $\sqrt{\frac{11}{70}} \doteq \sqrt{\frac{\underline{\quad}}{\underline{\quad}}}$
 $\doteq \frac{\underline{\quad}}{\underline{\quad}}$

7. Identify a decimal that has a square root between the two given numbers.

Check the answer.

a) 2 and 3

$2^2 = \underline{\quad}$ and $3^2 = \underline{\quad}$

So, any number between ____ and ____ has a square root between 2 and 3.

Choose ____.

Check: $\sqrt{\underline{\quad}} \doteq \underline{\quad}$

The decimal ____ is one possible answer.

b) 6.5 and 7.5

____ = ____ and ____ = ____

So, any number between ____ and ____ has a square root between 6.5 and 7.5.

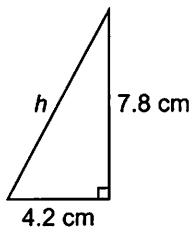
Choose ____.

Check: $\sqrt{\underline{\quad}} \doteq \underline{\quad}$.

The decimal ____ is one possible answer.

8. Find the length of the hypotenuse of each right triangle.

a)



$h^2 = \underline{\quad} + \underline{\quad}$

$h^2 = \underline{\quad} + \underline{\quad}$

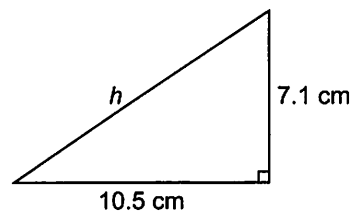
$h^2 = \underline{\quad}$

$h = \sqrt{\underline{\quad}}$

$h \doteq \underline{\quad}$

The length of the hypotenuse is about ____ cm.

b)



$h^2 = \underline{\quad} + \underline{\quad}$

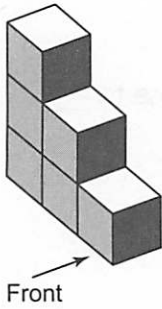
$h^2 = \underline{\quad} + \underline{\quad}$

$h^2 = \underline{\quad}$

$h = \sqrt{\underline{\quad}}$

$h \doteq \underline{\quad}$

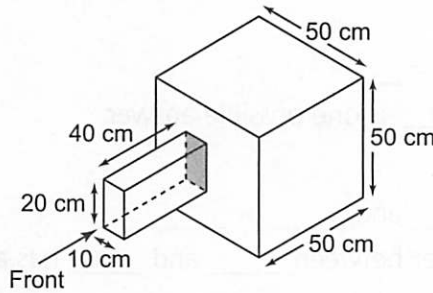
1.3 9. This object is made from 1-cm cubes. Find its surface area.



Matching Views	Diagram	Corresponding Area (cm ²)
_____ _____		_____
_____ _____		_____
_____ _____		_____
Total		_____

The surface area is _____ cm².

10. Calculate the surface area of this composite object.



Surface area of cube

Matching Faces	Diagram	Corresponding Area (cm ²)
____ / ____ ____ / ____ ____ / ____		6(____ × ____) = _____
Total		_____

The surface area is _____ cm².

Surface area of rectangular prism

Matching Faces	Diagram	Corresponding Area (cm ²)
____ / ____		_____
____ / ____		_____
____ / ____		_____
Total		_____

The surface area is _____ cm².

Area of overlap

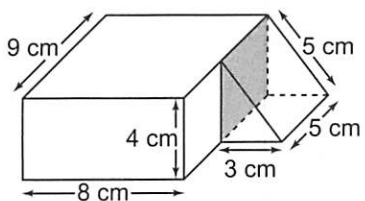
Diagram	Corresponding Area (cm ²)
	___ × ___ = ___

The area of overlap is ___ cm².

SA composite object = _____ + _____ - _____
 = _____ + _____ - _____
 = _____

The surface area of the composite object is _____ cm².

1.4 11. Find the surface area of this composite object.



Surface area of rectangular prism

Matching Faces	Diagram	Corresponding Area (cm ²)
_____		_____
_____		_____
_____		_____
_____		_____
Total		_____

The surface area is _____ cm².

Surface area of triangular prism

Matching Faces	Diagram	Corresponding Area (cm ²)
Triangular		_____

Rectangular		_____

Total		_____

The surface area is _____ cm².

Area of overlap

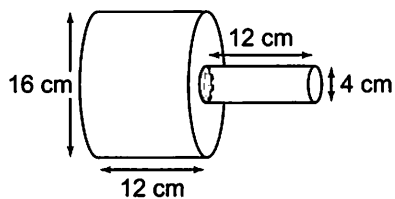
Diagram	Corresponding Area (cm ²)

The area of overlap is _____ cm².

SA = _____ + _____ - _____
 = _____ + _____ - _____
 = _____

The surface area of the composite object is _____ cm².

12. Find the surface area of this composite object.



The larger cylinder has diameter ___ cm, so its radius is ___ cm.

The smaller cylinder has diameter ___ cm, so its radius is ___ cm.

Surface area of smaller cylinder

Matching Faces	Diagram	Corresponding Area (cm ²)
Top Bottom		__ × __ × __ ÷ ____
Curved surface		__ × __ × __ × __ ÷ ____
Total		_____

The surface area is about _____ cm².

Surface area of larger cylinder

Matching Faces	Diagram	Corresponding Area (cm ²)
Top Bottom		__ × __ × __ ÷ ____
Curved surface		__ × __ × __ × __ ÷ ____
Total		_____

The surface area is about _____ cm².

Area of overlap

Diagram	Corresponding Area (cm ²)
	__ × __ ÷ ____

The area of overlap is about _____ cm².

Surface area of the composite object ÷ _____ + _____ - _____
 ÷ _____

The surface area is about _____ cm².