Linear Relations

What You'll Learn

- Use expressions and equations to write patterns and work with them.
- Use substitution to work with patterns to find more information.
- Graph and analyze linear relations.
- Use interpolation and extrapolation to gather more information from graphs.

Why It's Important

Patterns and linear relations are used by

- book printers, to quote the cost of a job
- managers, to plan for new hiring

Key Words

variable	discrete
expression	origin
equation	vertical
table of values	horizontal
relation	oblique
linear relation	interpolation
coordinates	extrapolation
coordinate grid	

▶ 4.1 Skill Builder

Algebraic Expressions

We can use a variable, such as n, to represent a number in an expression.

For example:

- 4 more than a number: $4 + n$, or $n + 4$
- 5 times a number: $5n$
- 2 less than a number: $n - 2$
- A number divided by 10: $\frac{n}{10}$

> A variable is a letter, and it is always written in italics.

To evaluate an expression, we replace a variable with a number.

Check

1. Find the value of each expression when $x = 3$.

a) $3x + 5$

$3(\underline{}) + 5 = \underline{} + 5$

$\phantom{3(\underline{xx}) + 5} = \underline{}$

b) $6 + 8x$

$6 + 8(\underline{}) = 6 + \underline{}$

$\phantom{6 + 8(\underline{xx})} = \underline{}$

2. Find the value of each expression when $n = 8$.

a) $8n - 4$

$8(\underline{}) - \underline{} = \underline{} - \underline{}$

$\phantom{8(\underline{xx}) - xx} = \underline{}$

b) $20 - 2n$

$\underline{} - 2(\underline{}) = \underline{} - \underline{}$

$\phantom{xx - 2(\underline{xx})} = \underline{}$

Relationships in Patterns

Here is a pattern made with toothpicks.

Figure 1   Figure 2   Figure 3

A pattern rule for the number of toothpicks is:

Start with ⬜. Add ⬜ each time.
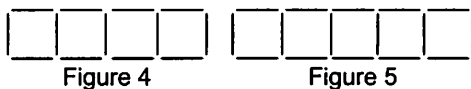
There is a pattern in the numbers as well.

Start with 4. Add 3 each time.

We can also show the pattern using a table of values.
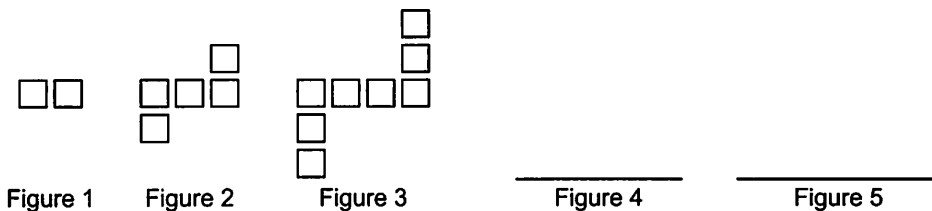
Figure Number	Number of Toothpicks
1	4
2	7
3	10

+3
+3

To extend the pattern, continue to add 3 each time:

Figure 4   Figure 5

Check

1. a) Draw the next 2 figures to extend each pattern.

i)

Figure 1   Figure 2   Figure 3   Figure 4   Figure 5

ii)

Figure 1   Figure 2   Figure 3   Figure 4   Figure 5
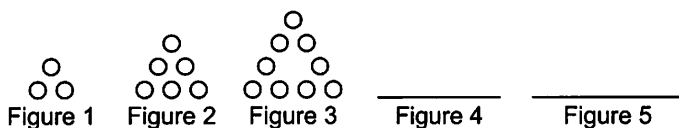
b) Complete each table of values to show each number pattern above.

i)

Figure Number	Number of Squares
1	2
2	___
3	___
___	___
___	___

ii)

Figure Number	Number of Dots
1	3
2	___
3	___
___	___
___	___

▶ 4.1 Writing Equations To Describe Patterns

FOCUS Use equations to describe and solve problems involving patterns.

Example 1 | Writing an Equation to Represent a Written Pattern

Here is a pattern made with sticks.

△ △▽ △▽△ △▽△▽ △▽△▽△
Figure 1   Figure 2   Figure 3   Figure 4   Figure 5
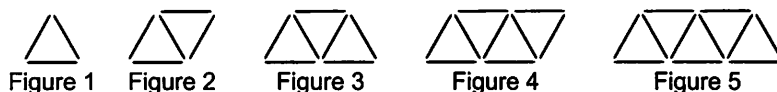
a) Write an equation that relates the number of sticks to the figure number.

b) What is the number of sticks in the 10th figure?

Solution

a) Record the number of sticks in each figure in a table.

As the figure number increases by 1, the number of sticks increases by 2. Repeated addition of 2 is the same as multiplication by 2.

Figure Number, f	Number of Sticks, n
1	3
2	5
3	7
4	9
5	11

So, the equation $n = 2f$ may represent the relationship.

Check whether the equation is correct.
When $f = 1$, $n = 2(1) = 2$
This is 1 less than 3.
So, add 1.
$2(1) + 1 = 3$

So, an equation is: $n = 2f + 1$

Figure Number, f	Number of Sticks, n
1	$2(1) + 1 = 3$
2	$2(2) + 1 = 5$
3	$2(3) + 1 = 7$
4	$2(4) + 1 = 9$
5	$2(5) + 1 = 11$

b) To find the number of sticks in the 10th figure, substitute $f = 10$ in the equation:
$n = 2f + 1$
$= 2(10) + 1$
$= 20 + 1$
$= 21$
There are 21 sticks in the 10th figure.

2f + 1 represents the number of sticks for any figure number f.

Check

1. For the table below:

	Number of Swings, s	Number of Hits, h	
	1	5	
	2	9	
	3	13	
	4	17	

a) Describe the number of hits in terms of the number of swings.
The number of hits is _____ times the number of swings, plus _____.

b) Write an equation to describe the relationship.
$h =$ _____ $s +$ _____

c) Use your equation to find h when $s = 10$.
$h =$ _____

_____

_____

| **Example 2** | **Writing an Equation to Represent a Situation** |

Teagan goes to a carnival.
The cost for a ride is
shown on a poster
at the entrance.

> Entrance Fee $5.00
> +
> $2.00 per ride

a) Write an equation that relates the total cost, C dollars,
to the number of rides, r.

b) Teagan goes on 4 rides. What is his total cost?

Solution

a) The cost is $5.00, plus $2.00 per ride.

That is, the cost is: $5.00 + 2.00 \times$ (number of rides)

An equation is: $C = 5.00 + 2.00r$

b) Use the equation: $C = 5.00 + 2.00r$

Substitute: $r = 4$

$$C = 5.00 + 2.00(4)$$
$$= 5.00 + 8.00$$
$$= 13.00$$

Teagan's total cost is $13.00.

Check

1. Marcel takes a summer job at a book packaging plant.
He gets paid $50 a day, plus $2 for every box packed.

a) Write an equation that relates the number of boxes packed to Marcel's pay for a day.

Let P represent his pay for one day, and let b represent the number of boxes packed.

$P = $ _____

b) Marcel packed 20 boxes one day. How much did he get paid?

$P = $ _____

$ = $ _____

$ = $ _____

Marcel got paid _____.

1. In each equation, find the value of T when $n = 6$.

a) $T = 8 + n$

$T = 8 + \rule{1cm}{0.4pt}$

$= \rule{2cm}{0.4pt}$

b) $T = 3n - 2$

$T = 3\rule{1cm}{0.4pt} - 2$

$= \rule{2cm}{0.4pt}$

$= \rule{2cm}{0.4pt}$

c) $T = 12n + 9$

$T = \rule{3cm}{0.4pt}$

$= \rule{2cm}{0.4pt}$

$= \rule{2cm}{0.4pt}$

d) $T = 7n + 3$

$T = \rule{3cm}{0.4pt}$

$= \rule{2cm}{0.4pt}$

$= \rule{2cm}{0.4pt}$

2. a) This pattern of dots continues. Draw the next 2 figures in the pattern.
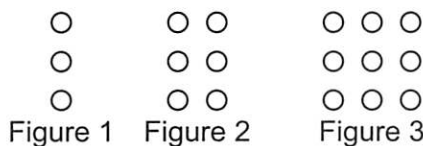
```
 O        O O       O O O
 O        O O       O O O
 O        O O       O O O
Figure 1  Figure 2  Figure 3    _____    _____
                                Figure 4    Figure 5
```

b) The pattern is represented in a table of values. Which expression below represents the number of dots in terms of the figure number?

Figure Number, f	Number of Dots, n
1	3
2	6
3	9
4	12
5	15

i) $2f$

ii) $3f$

iii) $-3f$

iv) $3f + 1$

_____

3. a) Look at the pattern of tiles below.
Draw the next 2 figures in the pattern.
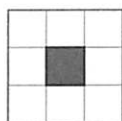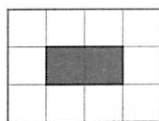
Figure 1   Figure 2   Figure 3   _____   _____
                                        Figure 4   Figure 5
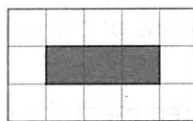
b) Complete the table below.

Number of Shaded Tiles, s	Number of White Tiles, w
1	8
2	____
____	____
____	____
____	____

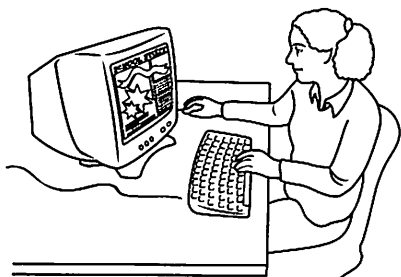**c)** Write an equation for the number of white tiles, w, in terms of the number of shaded tiles, s.

$w = $ ____ $s + $ ____

d) Use your equation to find w when $s = 25$.

$w = $ _____

$= $ _____

$= $ _____

When the number of shaded tiles is 25, there are ____ white tiles.

4. Anabelle is part of the yearbook committee. This year, the set-up cost to print yearbooks is $400, plus $3 for each yearbook printed.

a) Write an equation for the total cost in terms of the number of yearbooks printed.
Let C represent the total cost, and let n represent the number of yearbooks.

$C = $ _____ $ + $ _____

b) Anabelle takes 200 orders for yearbooks this year.
What is the total cost to the yearbook committee?

$C = $ _____

$= $ _____

$= $ _____

The total cost is $ _____ .

▶ 4.2 Skill Builder

The Coordinate Grid

Plot A(−2, 3) on a coordinate grid.
The first number in the ordered pair tells how far you move left or right on the horizontal axis.
The second number tells how far you move up or down on the vertical axis.
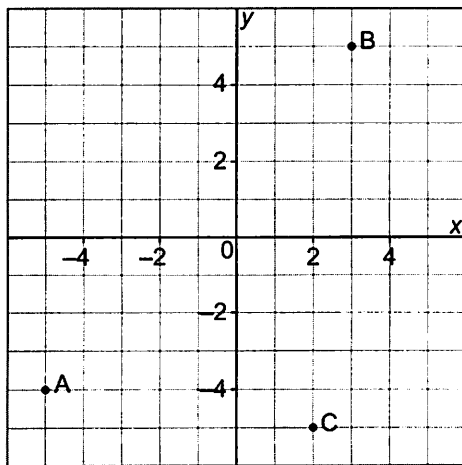
So, to plot A(−2, 3):
Move 2 squares left of the origin, then move 3 squares up.

−2 is the x-coordinate. 3 is the y-coordinate.
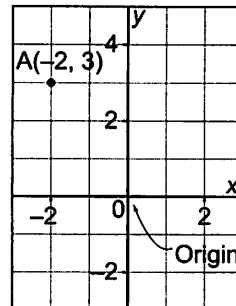
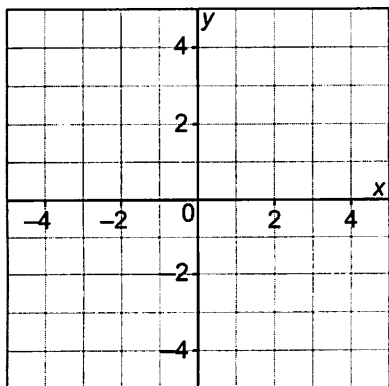

Check

1. What are the coordinates of points A, B, and C?

A(_____)
B(_____)
C(_____)

2. Graph these points on the coordinate grid.

A(−3, 0)   B(2, 4)   C(0, −3)

Graphing Relations

This table of values shows how $2n + 1$ relates to n.

Input, n	Output, $2n + 1$
1	3
2	5
3	7
4	9
5	11

+1 +2
+1 +2
+1 +2
+1 +2

The change in input is constant. The change in output is constant.

Graph of $2n + 1$ against n
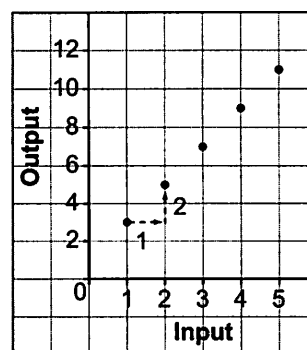
Use the data in the table to graph the relation.

The points lie on a straight line. This is a **linear relation.**
The graph also shows how $2n + 1$ relates to n.
On the graph, we see that each time the input increases by 1, the output increases by 2.

Check

1. a) Graph the data in this table of values.

Input	Output
0	2
1	5
2	8
3	11
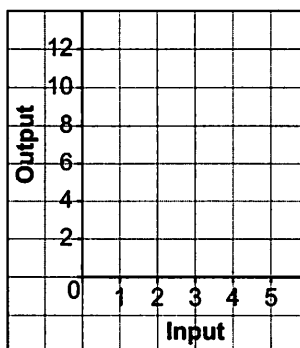
b) Is this a linear relation? Explain.

_____

_____

▶ 4.2 Linear Relations

FOCUS Analyze the graph of a linear relation.

A photographer charges $40 for a sitting fee, plus $20 per sheet of prints.
The charges are shown in the table of values and in the graph.

Number of Sheets, n	Cost, C ($)
0	40
1	60
2	80
3	100

Cost for Number of Sheets of Photos

We cannot order part of a sheet of prints.
So, the points in the graph are not joined with a line.
We say that the data are **discrete.**
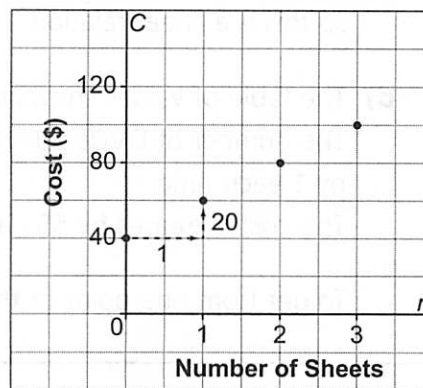
For different values of *n*, we get different values of *C*.
So the variable *C depends* on the value of the variable *n*.
When two variables are related in this way, they form a **relation.**

> **Linear Relation**
> When the graph of a relation is a straight line, it is called a **linear relation.**

Example 1 | **Graphing a Linear Relation from a Table of Values**

A popular DVD club allows members to purchase DVDs
at a reduced price according to the table of values.

a) Graph the data.
b) Should the points be joined?
Why or why not?
c) Is the relation linear? Explain.
d) Describe the patterns in the table.
How are these patterns shown
on the graph?

Number of DVDs Purchased, n	Cost, C ($)
1	20
2	25
3	30
4	35

Solution

a) Plot the points on a grid.

b) The points should not be joined because you cannot buy part of a DVD.

c) The points on the graph lie on a straight line, so this is a linear relation.

d) The table of values shows that:
The number of DVDs purchased increases by 1 each time.
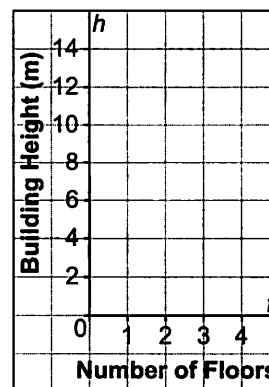The cost increases by $5 each time.
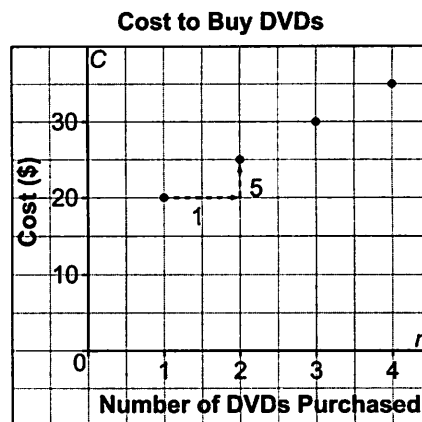
To get from one point to the next in the graph, move 1 unit right and 5 units up.

Cost to Buy DVDs

(graph: Cost ($) on vertical axis with marks at 10, 20, 30; Number of DVDs Purchased on horizontal axis with marks at 1, 2, 3, 4; points plotted rising by 5 each step, with steps labelled 1 and 5)

Check

1. a) Graph the data from the table of values.

Number of Floors, f	Building Height, h (m)
1	5
2	8
3	11
4	14

(blank graph: Building Height (m) on vertical axis with marks at 2, 4, 6, 8, 10, 12, 14; Number of Floors on horizontal axis with marks at 1, 2, 3, 4)

b) Is the relation linear? Explain.

_____

c) Should the points on the graph be joined with a line? Explain.

_____

Example 2 Graphing a Linear Relation from an Equation

a) Complete the table of values.

x	$y = 5 - x$
-2	
-1	
0	
1	
2	

b) Graph the relation represented by the data in the table of values.

c) Describe the patterns in the graph and in the table.

d) Is the relation linear? Explain.

Solution

a)

x	$y = 5 - x$
-2	$5 - (-2) = 7$
-1	$5 - (-1) = 6$
0	$5 - 0 = 5$
1	$5 - 1 = 4$
2	$5 - 2 = 3$

b)
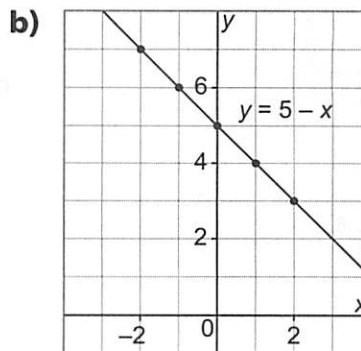
Since we can substitute any value for x, the points can be joined with a line.

c) Each point on the graph is 1 unit right and 1 unit down from the previous point. In the table, when x increases by 1, y decreases by 1.

d) This is a linear relation because its graph is a straight line.

Check

1. Complete the table of values.
Then, graph the relation.

x	$y = 4x - 2$
-1	-6
0	-2
1	_____
2	_____
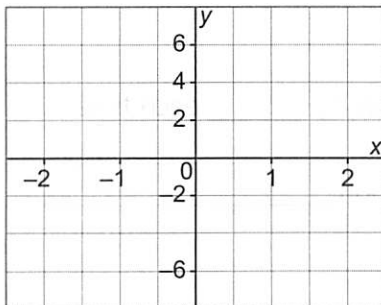
1. Which graphs represent a linear relation?

a)

_____

b)

_____

c)

_____

d)

_____
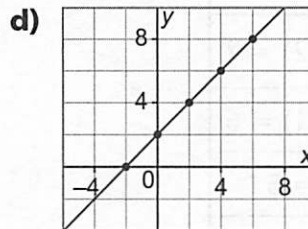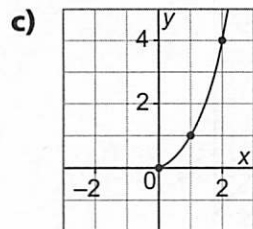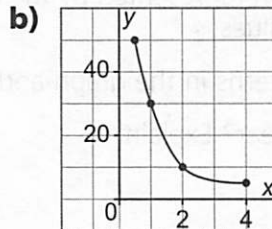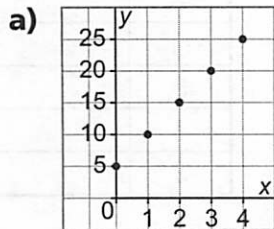
2. Describe the patterns in each table of values.
Does each table of values represent a linear relation?

a)

x	y
−3	6
−2	5
−1	4
0	3

x increases by _____ each time.
y decreases by _____ each time.
The relation is _____, because a constant change
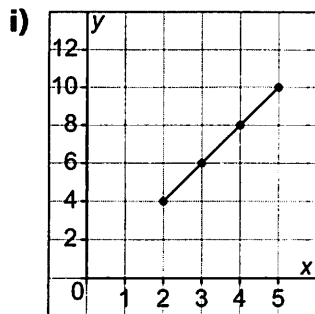in x produces a constant change in y.

b)

x	y
0	1
2	4
4	7
6	10

x increases by _____ each time.
y increases by _____ each time.
The relation is _____, because a constant change
in x produces a constant change in y.

c)

x	y
1	1
2	3
3	7
4	13

x increases by _____ each time.
y _____
The relation _____

3. Each graph and table of values represents a linear relation.

    a) Complete each table of values.

      i)

x	y
2	4
3	6
4	___
5	___

      ii)
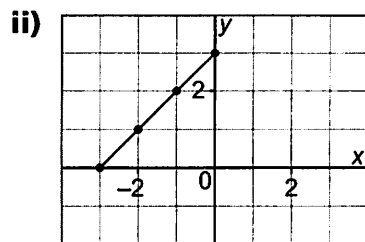
x	y
−3	0
−2	___
−1	2
0	___

    b) Describe the patterns in the table.

      i) When x increases by _____, y increases by _____.

      ii) When _____ increases by _____, _____ increases by _____.

    c) Describe the patterns in the graph.

      i) To get from one point to the next, move 1 unit right and _____ up.

      ii) To get from one point to the next, move _____ right and _____ up.

4. Complete the table of values for each linear relation, then graph it.

    a) $y = 4x$

x	y
−1	___
0	___
1	___
2	___
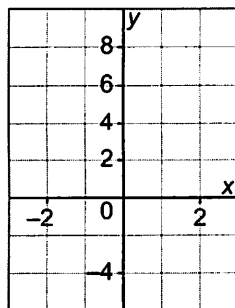
    b) $y = -3x$

x	y
−1	___
0	___
___	___
___	___
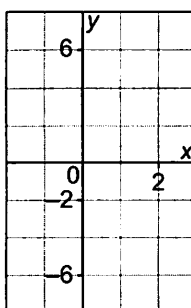
    c) $y = 1 - x$

x	y
0	___
___	___
___	___
___	___
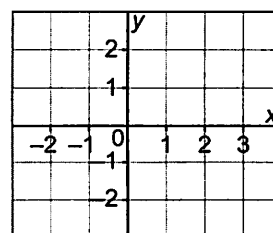
5. Complete the table of values.
Graph the data.
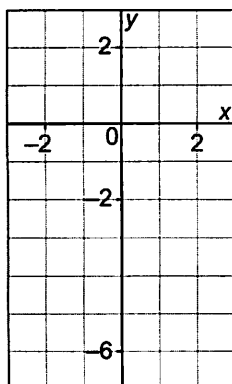
x	$y = 2x - 4$
−1	−6
0	−4
1	_____
2	_____

6. For special events, a bowling alley charges a set fee plus a fee for each hour bowled.

Bowling Costs

Hours, h	Cost, C ($)
1	40
2	50
3	60
4	70

a) Graph the data.

Bowling Costs

Does it make sense to join the points on the graph? Explain.

_____

_____

b) Is this a linear relation? Why?

_____

_____

c) Describe the pattern in words and using an equation.
When h increases by _____, C increases by _____.
$C =$ _____ $h + 30$

▶ 4.3 Skill Builder

Solving Equations

Solving an equation means finding a value of the variable that makes the equation true.
To solve the equation $3x - 2 = 7$, find the value of x so that the left and the right side of
the equation are balanced.

$3x - 2 = 7$	Add 2 to both sides to isolate x.
$3x - 2 + 2 = 7 + 2$	
$3x = 9$	Divide each side by 3.
$\dfrac{3x}{3} = \dfrac{9}{3}$	
$x = 3$	

To verify the equation, substitute $x = 3$ in the original equation.

Left side: $3(3) - 2$ Right side $= 7$
$\qquad\quad = 9 - 2$
$\qquad\quad = 7$

Since the left side equals the right side, the solution is correct.

Check

1. Solve each equation.

a) $2x + 3 = 11$

$2x + 3 -$ _____ $= 11 -$ _____

_____

_____

_____

Check your solution.

b) $3 - 2x = -9$

_____

_____

_____

_____

▶ 4.3 Another Form of the Equation for a Linear Relation

FOCUS Recognize the equations of horizontal, vertical, and oblique lines, and graph them.

Example 1 | Graphing and Describing Vertical Lines

For the equation $x = 4$:

a) Draw the graph.

b) Describe the graph.

Solution

$x = 4$

a) For any value of y, x is always 4.
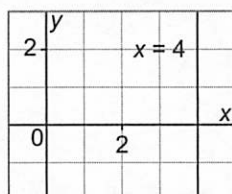
The vertical line intersects the x-axis at 4.

b) The graph is a vertical line.
Every point on the graph has x-coordinate 4.

When the equation is $x =$ a constant, the graph is a **vertical line**.

Check

1. For the equation $x = 1$:
Draw the graph.
Then describe the graph.
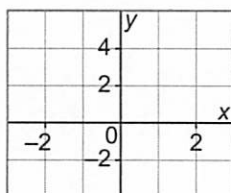
The graph is a _____ line.
Every point on the graph has _____-coordinate _____.

Example 2 | **Graphing and Describing Horizontal Lines**

For the equation $y + 1 = 0$:

a) Draw the graph.

b) Describe the graph.

Solution

$y + 1 = 0$ Solve for y. Subtract 1 from each side.

$y + 1 - 1 = 0 - 1$

$y = -1$

a)
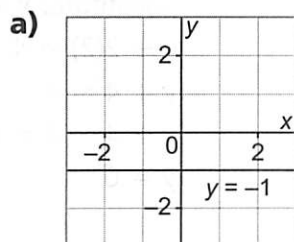
The horizontal line intersects the y-axis at -1.

b) The graph is a horizontal line.

Every point on the graph has y-coordinate -1.

When the equation is $y =$ a constant, the graph is a **horizontal line**.

Check

1. For the equation $y = 3$:

a) Draw the graph.

b) Describe the graph.

a)
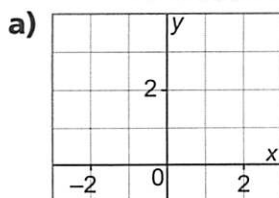
b) The graph is a _____.

Every point on the graph has _____-coordinate _____.

Example 3 | **Graphing an Equation**

For the equation $y + 2x = 4$:

    a) Make a table of values for $x = -2$, 0, and 2.

    b) Graph the equation.

Solution

    a) $y + 2x = 4$

      Substitute each value of x, then solve for y.

Substitute: $x = -2$	Substitute: $x = 0$	Substitute: $x = 2$
$y + 2(-2) = 4$	$y + 2(0) = 4$	$y + 2(2) = 4$
$y - 4 = 4$	$y + 0 = 4$	$y + 4 = 4$
$y - 4 + 4 = 4 + 4$	$y = 4$	$y + 4 - 4 = 4 - 4$
$y = 8$		$y = 0$

x	y
-2	8
0	4
2	0

    b) Plot the points on a grid, and join the points.

      The graph is an **oblique** line.
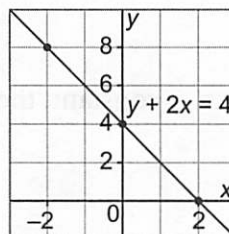
Oblique means slanted.

Check

1. Do not graph the equations.

Does each equation describe a horizontal line, a vertical line, or an oblique line?

    a) $y = 4$ _____

    b) $y = 3x - 2$ _____

    c) $x = -1$ _____

    d) $2x + y = -6$ _____

1. Which equation describes each graph?

i) $y = 4$ **ii)** $x = 2$ **iii)** $y = 2$ **iv)** $x = 4$

a)

b)

_____   _____
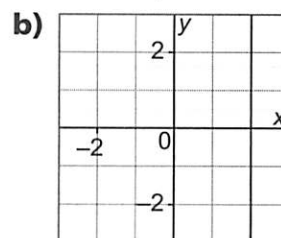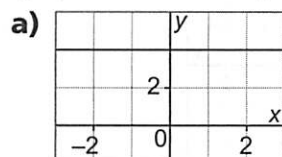
2. Write an equation to describe each line.

a)

b)

c)
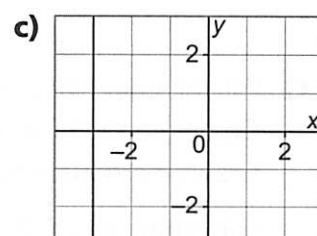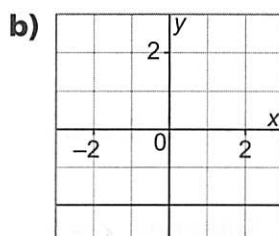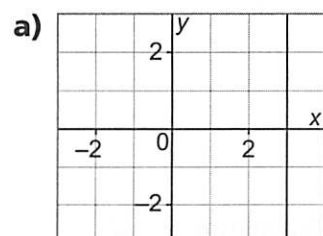
$x = $ _____ $y = $ _____   _____

3. a) Is each line vertical or horizontal?

   i) $x = -1$ **ii)** $y = -4$

   _____   _____

b) Graph each line. Describe the graph.

   i)

   Every point on the graph has
   x-coordinate _____.

   ii)
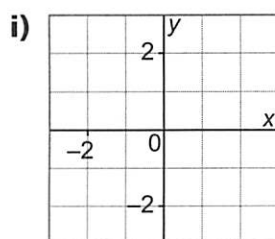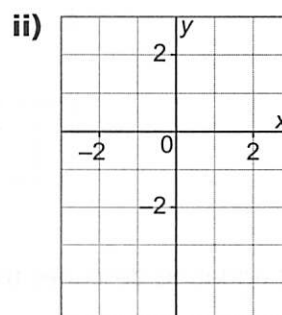
   Every point on the graph has
   _____-coordinate _____.

4. a) Does each equation describe a vertical line, a horizontal line, or an oblique line?

i) $x + 3 = -1$ **ii)** $1 + y = 0$ **iii)** $x + 2y = 8$

_____   _____   _____

b) Graph the first 2 equations in part a.

i) $x + 3 = -1$

_____

_____
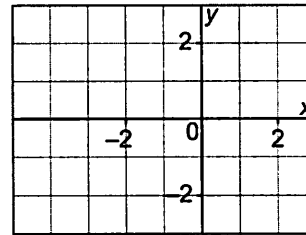
ii) $1 + y = 0$

_____

_____
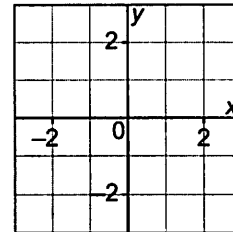
c) For the equation $x + 2y = 8$:
Complete the table of values for $x = -2$, 0, and 2.
Graph the equation.

Substitute: $x = -2$ Substitute: $x = 0$ Substitute: $x = 2$

_____ $+ 2y = 8$ _____ $+ 2y = 8$ _____ $+ 2y = 8$

_____   _____   _____

_____   _____   _____

_____   _____   _____

_____   _____   _____

x	y
-2	__
0	__
2	__
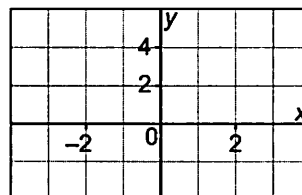
5. a) Explain why this equation describes the graph below.
$y + 3 = 0$

_____

_____

This is a _____ line, with _____-coordinate _____,
which matches the graph.
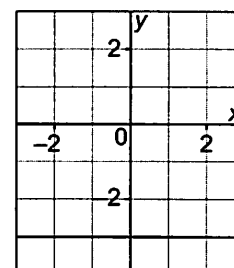
Can you ...

- Use equations to describe and solve problems involving patterns?
- Graph a linear relation?
- Recognize the equations of horizontal, vertical, and oblique lines, and graph them?

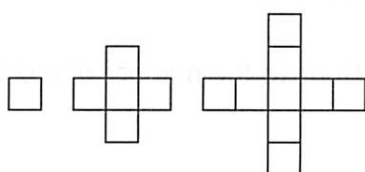**4.1** **1.** This pattern of squares continues. Draw the next 2 figures in the pattern.

Figure 1   Figure 2   Figure 3   Figure 4   Figure 5

a) Complete the table of values below.

Figure Number, n	Number of Squares, s
1	1
2	5
3	——
4	——
5	——

+1   +4
+1   +4
+1   +4
+1   +4

b) What patterns do you see?

The figure number increases by _____ each time.
The number of squares increases by _____ each time.

c) Describe how the number of squares relates to the figure number.

The number of squares is _____ times the figure number, less _____.

d) Write an equation for this pattern.

$s =$ _____ $n -$ _____

2. The pattern in the table of values continues. Complete the table.

Number of Red Buttons, r	Number of Blue Buttons, b
2	10
3	13
4	16
5	19
_____	_____
_____	_____

a) What patterns do you see? The number of red buttons increases by _____ each time. The number of blue buttons increases by _____ each time.

b) Write an equation that relates the number of blue buttons to the number of red buttons.
$b =$ _____ $r +$ _____

4.2 **3.** A home service provider charges for the service according to the table of values.

a) Graph the data.

Home Service Charges
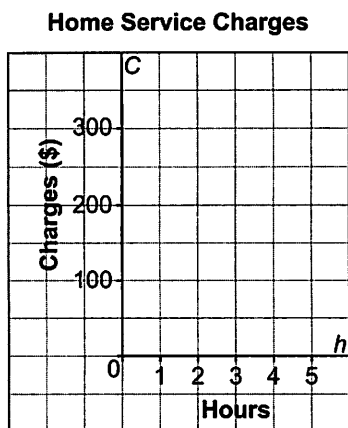
Home Service Charges

Hours, h	Charges, C ($)
0	60
1	150
2	240
3	330

b) Is this an example of a linear relation? Why?

_____

c) Describe the patterns in the table.
As h increases by _____, C increases by _____.

d) How is the pattern shown in the graph?
On the graph, to get from one point to the next, move 1 unit right, and _____ up.

e) Write an equation for this pattern.
$C =$ _____ $h +$ _____

4. a) This table of values represents a linear relation.
Graph the data.

x	y
−2	−2
−1	0
0	2
1	4
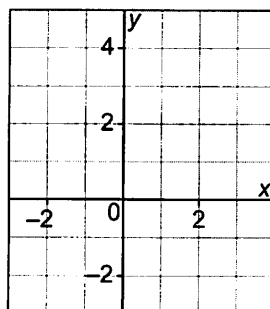
b) How do the patterns in the graph relate to the patterns in the table?
In the table, as *x* increases by _____, *y* increases by _____.
On the graph, to get from one point to the next,
move 1 unit right and _____ up.

c) Write an equation for this pattern.
$y = $ _____ $x + $ _____

4.3 **5.** Does each equation describe a horizontal line, a vertical line, or an oblique line?

a) $x + 4 = 0$

_____

b) $y = -6$

_____

c) $x + y = 2$

_____

d) $2y = 4$

_____

6. Write an equation to describe each line.

a)

_____

b)

_____

c)

_____
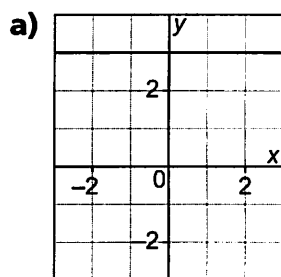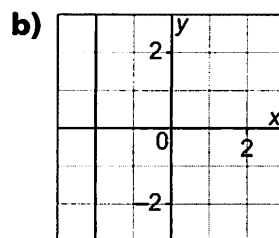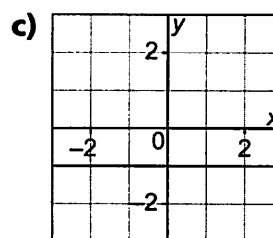
▶ 4.4 Matching Equations and Graphs

FOCUS Match equations and graphs of linear relations.

Example 1 | Matching Equations with Graphs

Match each graph on the grid with its equation.

$y = x$

$y = -x$
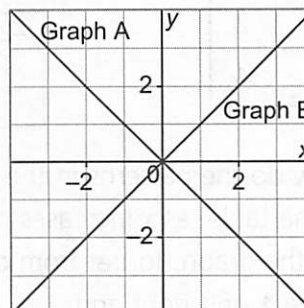
Solution

Substitute $x = -1$, $x = 0$, and $x = 1$ in each equation.

$y = x$

x	y
−1	−1
0	0
1	1

We chose to use x-values of −1, 0, and 1 because they're often easy to substitute.

Points $(−1, −1)$, $(0, 0)$, and $(1, 1)$ lie on Graph B.

So, $y = x$ matches Graph B.

$y = -x$

x	y
−1	1
0	0
1	−1

Points $(−1, 1)$, $(0, 0)$, and $(1, −1)$ lie on Graph A.

So, $y = -x$ matches Graph A.

Check

1. Which equation describes the graph at the right?

$y = x + 2$ $y = x - 2$

x	y = x + 2
0	y = 0 + 2 = _____
1	y = _____ + 2 = _____
2	y = _____ + 2 = _____

x	y = x − 2
0	y = _____ − 2 = _____
1	y = _____ = _____
2	y = _____ = _____
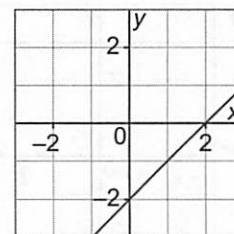
Points (_____), (_____), and (_____) do not lie on the graph.

Points (_____), (_____), and (_____) lie on the graph.

So, the equation $y = $ _____ describes the graph.

Example 2 | **Identifying a Graph Given Its Equation**

Which graph on this grid has the equation $y = x - 1$?
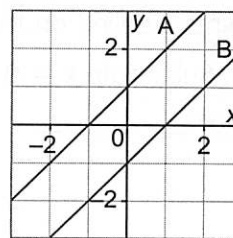
Solution

Pick 2 points on each graph and check if their coordinates satisfy the equation.

For Graph A, use: C(-1, 0) and D(0, 1)
Check if C(-1, 0) satisfies the equation $y = x - 1$.
Substitute $x = -1$ and $y = 0$ in $y = x - 1$
Left side: $y = 0$ Right side: $x - 1 = (-1) - 1$
                                                    $= -2$

Since C does not work, we do not have to check for D.

The left side does not equal the right side.
So, Graph A does not have equation $y = x - 1$.

Verify that the other graph does match the equation.
For Graph B, use: E(0, -1) and F(1, 0)
Check if E(0, -1) satisfies the equation $y = x - 1$.
Substitute $x = 0$ and $y = -1$ in $y = x - 1$
Left side: $y = -1$ Right side: $x - 1 = 0 - 1$
                                                    $= -1$

The left side does equal the right side.
So, E(0, -1) lies on the line represented by $y = x - 1$.

Check if F(1, 0) satisfies the equation $y = x - 1$.
Substitute $x = 1$ and $y = 0$ in $y = x - 1$
Left side: $y = 0$ Right side: $x - 1 = 1 - 1$
                                                    $= 0$

The left side does equal the right side.
So, F(1, 0) lies on the line represented by $y = x - 1$.
So, Graph B has equation $y = x - 1$.

Check

1. Show that this graph has equation $y = 2x + 1$.

Use the points labelled on the graph.

For A(0, 1): Substitute $x = 0$ and $y = 1$ in $y = 2x + 1$.

Left side: $y =$ _____   Right side: $2x + 1 =$ _____
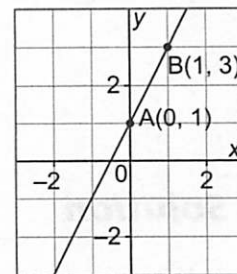
$=$ _____

$=$ _____

For B(1, 3): Substitute $x =$ _____ and $y =$ _____ in $y = 2x + 1$.

Left side: $y =$ _____   Right side: $2x + 1 =$ _____
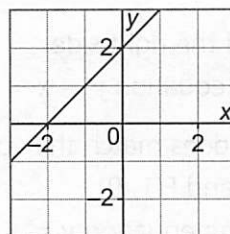
$=$ _____

$=$ _____

Practice

1. Show that the equation $y = x + 2$ matches the graph.

Fill in the table of values.

x	$y = x + 2$
-2	$y = -2 + 2 =$ ____
-1	$y =$ ___ $+ 2 =$ ____
0	$y =$ _____ $=$ ____

From the table:

Points (_____), (_____), and (_____) lie on the graph.

So, $y = x + 2$ matches the graph.

2. Match each equation with a graph.

$y = 3x$ $y = -3x$

Fill in the tables of values.

x	$y = 3x$
-1	$y = 3($___$) =$ ___
0	$y = 3($___$) =$ ___
1	$y = 3($___$) =$ ___

x	$y = -3x$
-1	$y = -3($___$) =$ ___
0	$y =$ ___$($___$) =$ ___
1	$y =$ _____ $=$ ___
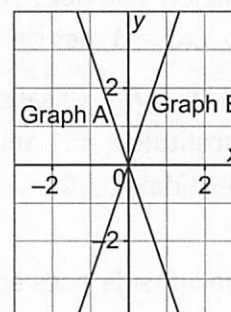
From the tables:

$y = 3x$ has points (_____), (_____), and (_____).

These points lie on Graph _____.
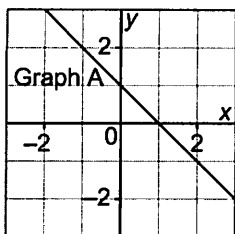
So, $y = 3x$ matches Graph _____.

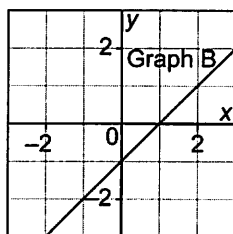$y = -3x$ has points (_____), (_____), and (_____).

These points lie on Graph _____.

So, $y = -3x$ matches Graph _____.

3. Match each equation with a graph.

$y = 1 - x$ $y = x - 1$

Graph A

Graph B

Fill in the tables of values.

x	$y = 1 - x$
-1	$y = 1 - (\underline{}) = \underline{}$
0	$y = 1 - \underline{} = \underline{}$
1	$y = 1 - \underline{} = \underline{}$

x	$y = x - 1$
-1	$y = \underline{} = \underline{}$
0	$y = \underline{} = \underline{}$
1	$y = \underline{} = \underline{}$

From the tables:

$y = 1 - x$ has points (_____), (_____), and (_____).

These points lie on Graph _____.

So, $y = 1 - x$ matches Graph _____.

$y = x - 1$ has points (_____), (_____), and (_____).

These points lie on Graph _____.

So, $y = x - 1$ matches Graph _____.

4. Which graph has equation $y = x - 3$?

For C(-3, 0):

Left side: $y = $ _____   Right side: $x - 3 = $ _____

                                                $= $ _____

The left side _____ equal the right side.

_____

For E(0, -3):

Left side: $y = $ _____   Right side: $x - 3 = $ _____

                                                $= $ _____

The left side _____ the right side.

For F(3, 0):

Left side: $y = $ _____   Right side: $x - 3 = $ _____

                                                $= $ _____

_____

So, Graph _____ has equation $y = x - 3$.
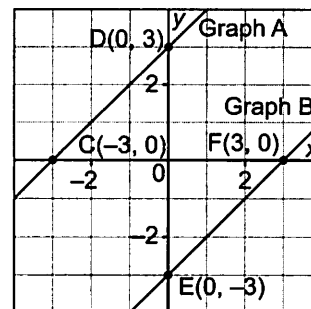
4.5 Using Graphs to Estimate Values

FOCUS Use interpolation and extrapolation to estimate values on a graph.

When we estimate values between 2 given data points on a graph of a linear relation, we use **interpolation**.

Example 1 | Using Interpolation to Solve Problems

This graph shows the distance travelled by Bobbie's family on a trip from Calgary to Moose Jaw. How long did it take his family to travel 320 km?
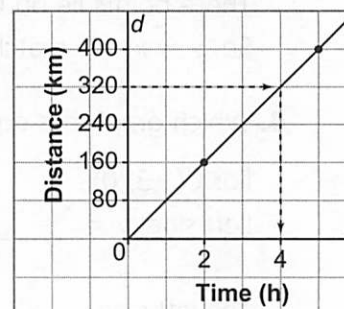
Bobbie's Family Trip
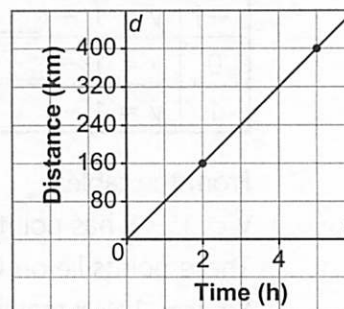
Solution

To find how long it took to travel 320 km:
- Locate the point on the vertical axis that represents 320 km.
- Draw a horizontal line to the graph.
- Then draw a vertical line from the graph to the horizontal axis.

Read the value where the vertical line meets the horizontal axis.

It took about 4 h to travel 320 km.

Bobbie's Family Trip

We could follow the same process to find that, after 3 h, the family has travelled about 240 km.

Check

1. Use the graph to find the following values.

Cost of Fuel

a) The cost of 15 L of fuel.
About $_____.

b) The quantity of fuel that can be purchased for $10.
About _____ L.
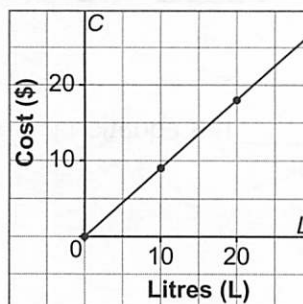
When we extend a graph of a linear relation to estimate values that lie beyond the graph, we use **extrapolation.**

Example 2   Using Extrapolation to Solve Problems

On his family trip from Calgary to Moose Jaw, Bobbie wants to predict how long it will take to travel 640 km.

Bobbie's Family Trip
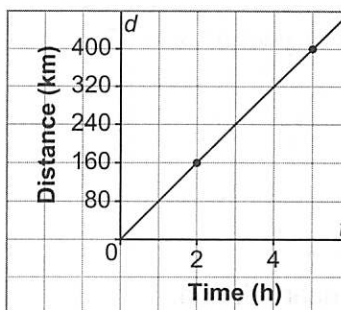
*We assume that Bobbie's family will continue to travel at the same average speed.*

Solution

Since the relation appears to be linear, we can extend the graph.

- Locate the point on the vertical axis that represents 640 km.
- Draw a horizontal line to the graph.
- Then draw a vertical line from the graph to the horizontal axis.

Read the value where the vertical line meets the horizontal axis. It will take about 8 h to travel 640 km.

Bobbie's Family Trip
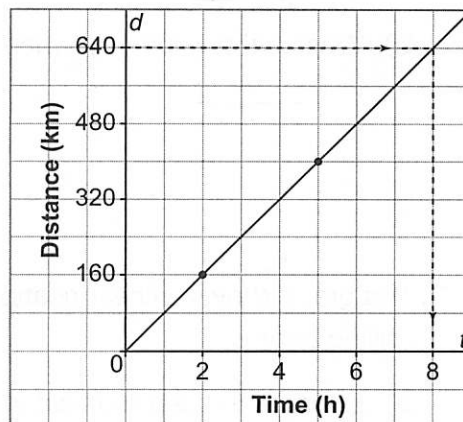
Check

1. Use the graph to find the cost of 30 L of fuel.

Cost of Fuel
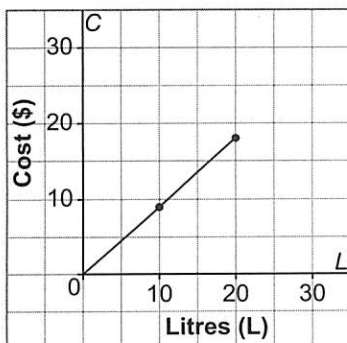
1. Use this graph of a linear relation.

    a) What is the value of x when $y = 3$?

        $x =$ _____

    b) What is the value of y when $x = 1$?

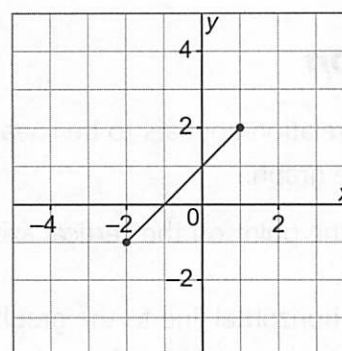 $y =$ _____
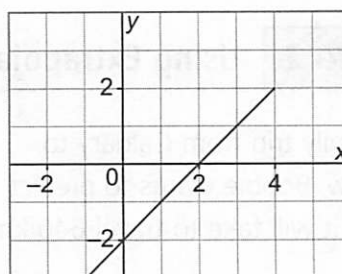
2. This graph shows a linear relation.

    a) What is the value of x when $y = 4$?

        $x =$ _____

    b) What is the value of y when $x = -4$?

        $y =$ _____

3. This graph shows a linear relation for different drilling depths.

    a) Estimate the depth drilled in 1 day.

        About _____ m

    b) Estimate the time taken to drill to a depth of 750 m.

        About _____ days

    c) Estimate the depth that will be drilled in 3 days.

        About _____ m

    d) Estimate the time it will take to drill 2000 m.

        About _____ days

Drilling Depths
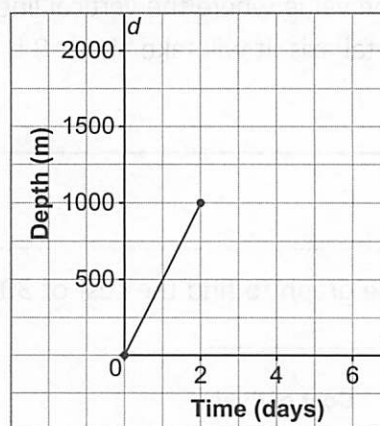
A Graphing Perspective
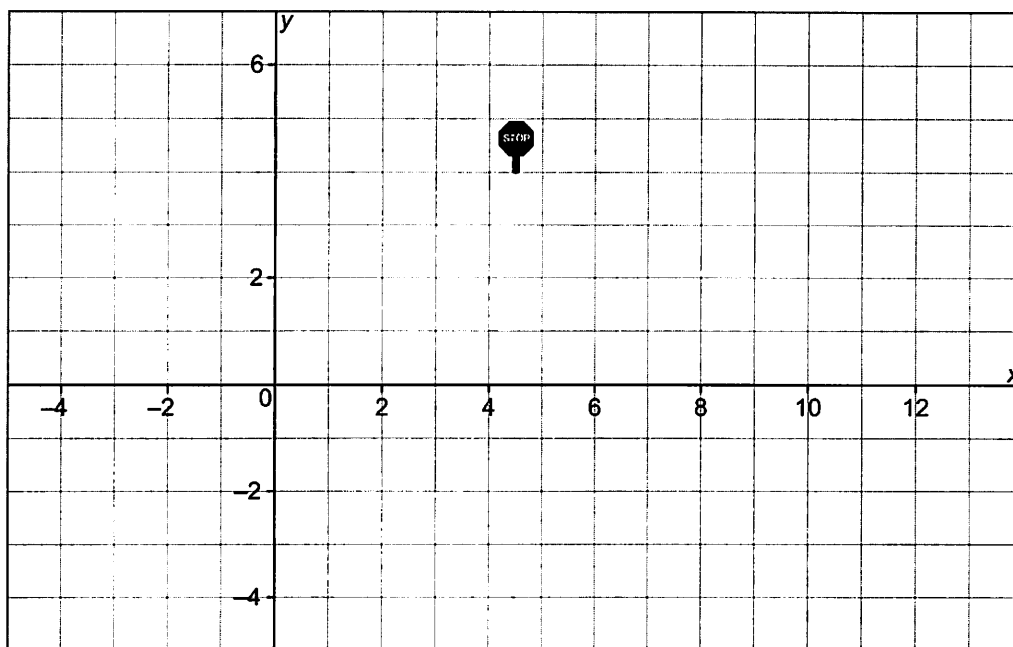
1. On the grid, plot the lines represented by:

a) $y = 4$

b) $x = y$

x	y
−4	___
0	___
4	___

c)

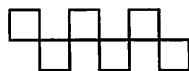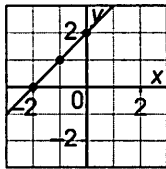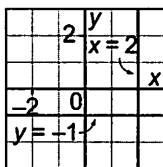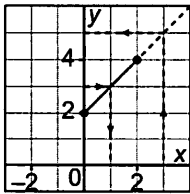| x	y
5	4
9	0
13	−4
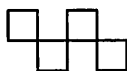
2. Plot (4.5, 4) and (4.5, 3), and join the points with a line.

3. Plot (4.5, 2) and (4.5, 0), and join the points with a line.

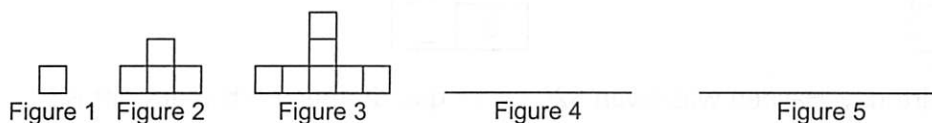**4.** Plot (4.5, −1) and (4.5, −4), and join the points with a line.

What do you see?

_____

Skill	Description	Example
Generalize a pattern	Recognize and extend a pattern using a drawing and a table of values. Describe the pattern. Write an equation for the pattern.	 Figure 1 Figure 2 Figure 3 **Figure Number, _n_** / **Figure Value, _v_** 1 / 2 2 / 4 3 / 6 As the figure number increases by 1, the figure value increases by 2. The pattern is: multiply the figure number by 2 to get the figure value. An equation is: $v = 2n$
Linear relations	The points on the graph of a linear relation lie on a straight line. To graph a linear relation, create a table of values first. In a linear relation, a constant change in x produces a constant change in y.	$\begin{array}{c\|c} x & y \\ \hline -2 & 0 \\ -1 & 1 \\ 0 & 2 \end{array}$ As x increases by 1, y increases by 1.
Horizontal and vertical lines	A vertical line has equation $x = a$ A horizontal line has equation $y = b$	The graph of $x = 2$ is a vertical line. Every point on the line has x-coordinate 2. The graph of $y = -1$ is a horizontal line. Every point on the line has y-coordinate -1.
Interpolation and extrapolation	When we estimate values between 2 given points on a graph, we use interpolation. When we estimate values beyond given points on a graph, we use extrapolation.	When $y = 3, x = 1$ Extend the graph to find that, when $x = 3, y = 5$

4.1 **1.** This pattern continues.

a) Draw the next 2 figures in the pattern.

Figure 1   Figure 2   Figure 3   _____ Figure 4   _____ Figure 5

b) Complete the table of values.

Figure Number, n	Number of Squares, s
1	1
2	4
3	7
_____	_____
_____	_____

c) Describe the patterns in the table.
The figure number increases by _____ each time.
The number of squares increases by _____ each time.

d) Write an equation that relates the number of squares to the figure number.
$s =$ _____ $n -$ _____

e) What is the number of squares in figure 10?
When $n = 10$:
$s =$ _____ $- 2 =$ _____ $- 2 =$ _____
There are _____ squares in figure 10.

2. The pattern in this table of values continues.

a) Complete the table.

b) Which expression below represents the number of squares in terms of the figure number? _____

i) $5n$

ii) $5n - 4$

iii) $n + 4$

iv) $n - 4$

Figure Number, n	Number of Squares, s
1	5
2	6
3	7
4	___
5	___

3. Complete each table of values.

a) $y = x + 1$

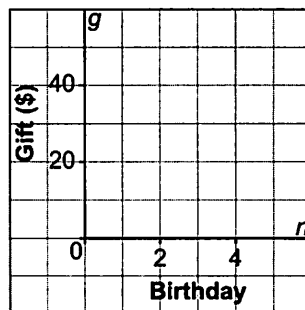| x	y
1	__
2	__
3	__
4	__

b) $y = x - 1$

x	y
2	__
4	__
6	__
8	__

4. On his first birthday, Hayden was given $20 by his grandfather. Each year's gift is $10 more than the year before. The data is given in the table below.

Grandfather's Gifts

Birthday, n	Gift, g ($)
1	20
2	30
3	40
4	50

a) Graph the data.

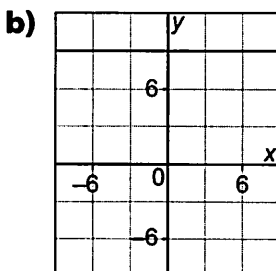**b)** Is the graph linear? Explain your thinking.
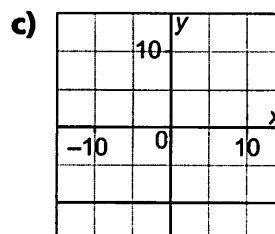The points _____, so the graph is _____.

c) Should the points be joined? Explain why or why not.

_____

_____

d) How are the patterns in the table shown in the graph? In the table, as the birthday increases by ____, the gift value increases by ____. Each point on the graph is _____ and _____ from the previous point.

5. Write an equation to describe each line.

a)

_____

b)

_____

c)

_____
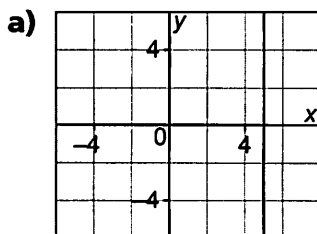
6. Does each equation represent a horizontal line, a vertical line, or an oblique line?

a) $x = 2$　　　b) $y = 2x + 2$　　　c) $y = 3$　　　d) $x = -1$

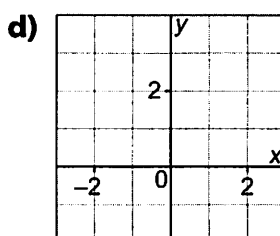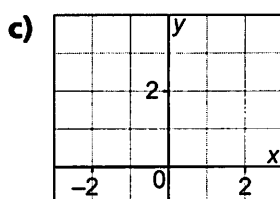_____　　　_____　　　_____　　　_____

Draw a graph for each equation above.

a)

x	y
___	___
___	___
___	___

b)

c)

d)

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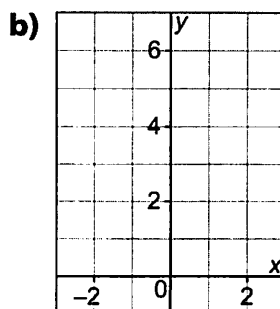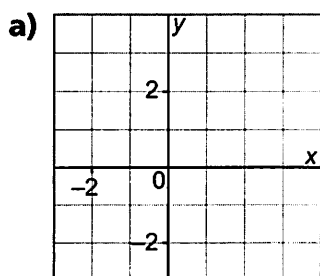
7. Which equation describes the graph?

$y = 2x$ or $y = -2x$

Fill in the tables of values.

x	y = 2x
−1	2(___) = ___
0	2(___) = ___
1	2(___) = ___

x	y = −2x
−1	−2(___) = ___
0	___ = ___
1	___ = ___
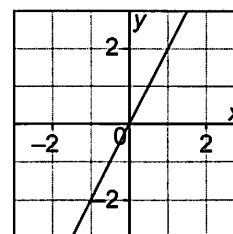
From the tables:

$y = 2x$ has points (_____), (_____), and (_____).

$y = -2x$ has points (_____), (_____), and (_____).

The graph passes through the points (_____), (0, 0), and (_____).

So, $y =$ _____ describes the graph.

8. Which graph represents the equation $x - y = 2$?

For A(−2, 0):

Left side: $x - y =$ _____   Right side: _____

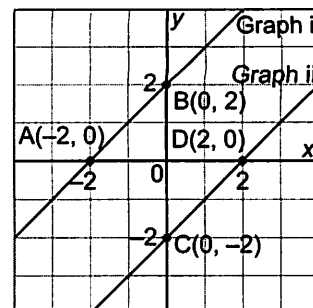   = _____

The left side _____ equal the right side.

_____

For C(0, −2):

Left side: $x - y =$ _____   Right side: _____

            = _____

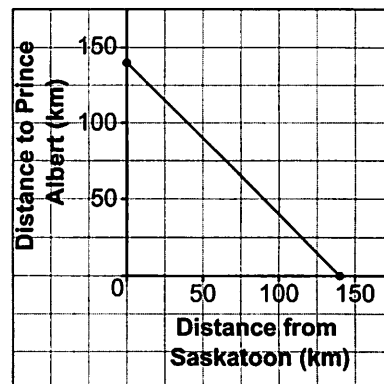The left side _____ the right side.

For D(2, 0):

_____

_____

_____

So, Graph _____ has equation $x - y = 2$.

4.5 **9.** This graph shows Emma's and Julianna's journey from Saskatoon to Prince Albert.

When Emma and Julianna have travelled 100 km, about how far do they still have to go?

_____

Journey from Saskatoon to Prince Albert

10. This graph represents a linear relation.

  a) Estimate the value of y when:

    i) $x = 0$ **ii)** $x = 1$

    _____   _____

  b) Estimate the value of x when:

    i) $y = 4$ **ii)** $y = -2$

    _____   _____